



# Lean Six Sigma Statistical Tools, Templates & Monte Carlo Simulation in Excel

## What's New in SigmaXL<sup>®</sup> Version 9

### Part 3 of 3: Control Charts for Autocorrelated Data



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Webinar December 10, 2020

# Agenda

- Introduction
- Autocorrelation
- Example 1: Chemical Process Concentration
- Simple Exponential Smoothing (EWMA)
- Example 2:  $\ln(\text{Monthly Airline Passengers-Modified})$

# Agenda

- Error, Trend, Seasonal (ETS) Exponential Smoothing models
- Autoregressive Integrated Moving Average (ARIMA) models
- ARIMA with Predictors
  - Analyze control chart outlier versus shift
  - Example 3: Electricity Demand with Temperature and Work Day Predictors
- Questions/References

# Introduction

- Statistical process control for autocorrelated processes typically use the EWMA (Exponentially Weighted Moving Average) one-step-ahead forecast model.
- The time series model forecasts the motion in the mean and an Individuals control chart is plotted of the residuals to detect assignable causes.

# Introduction

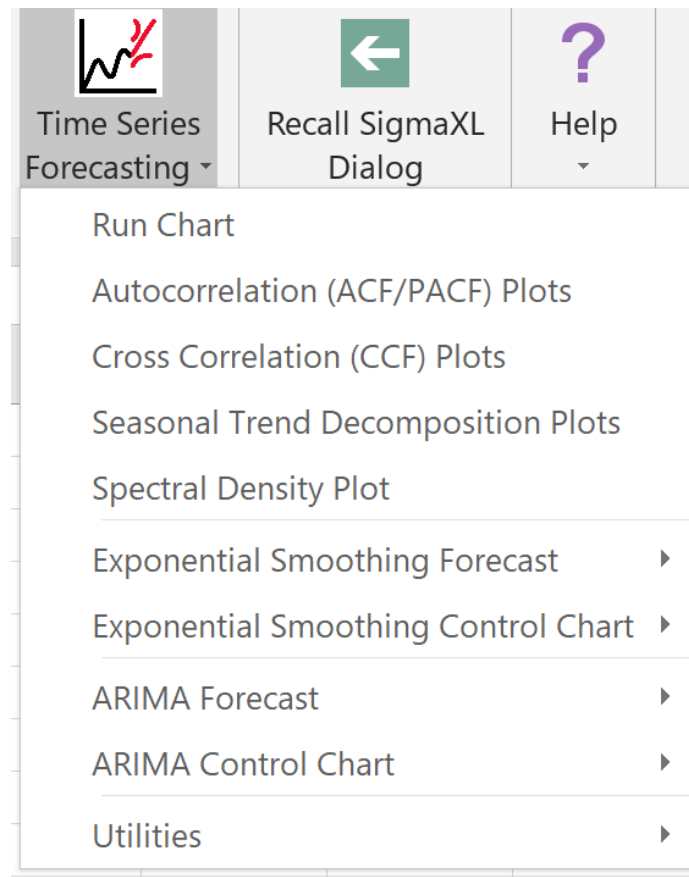
- Failure to account for the autocorrelation will produce limits that are too narrow resulting in excessive false alarms, or limits that are too wide resulting in misses.
- The challenge with this approach is that if there is seasonality or negative autocorrelation in the data, the user needs an advanced level of knowledge in forecasting methods to pick the correct model, e.g., Seasonal Exponential Smoothing models or Seasonal Autoregressive Integrated Moving Average (ARIMA) models are required.

# Introduction

- We will review simple exponential smoothing/EWMA, then introduce recent developments in time series forecasting that use automatic model selection to accurately pick the time series model that produces a minimum forecast error.
- An accurate forecast for your time series means the residuals will most often have the right properties to correctly apply a control chart, thus leading to an improved control chart with reduced false alarms and misses.

# SigmaXL Version 9

## Time Series Forecasting Menu



# Autocorrelation

- Just as correlation measures the extent of a linear relationship between two variables, autocorrelation (AC) measures the linear relationship between lagged values of data.
- A plot of the data vs. the same data at lag  $k$  will show a positive or negative trend. If the slope is positive, the AC is positive; if there is a negative slope, the AC is negative.
- The Autocorrelation Function (ACF) formula is:

$$r_k = \frac{\sum_{t=k+1}^T (y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^T (y_t - \bar{y})^2}$$

where  $T$  is length of the time series [4].

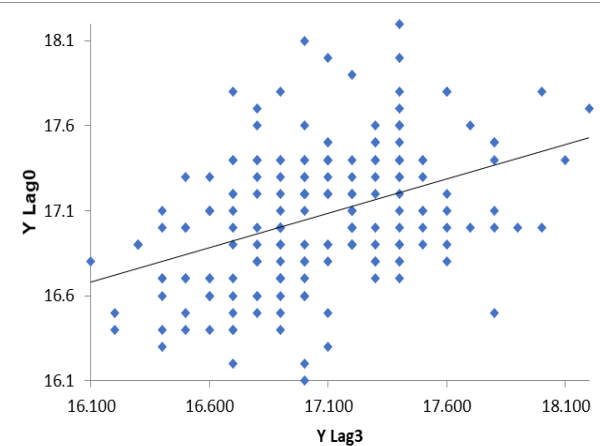
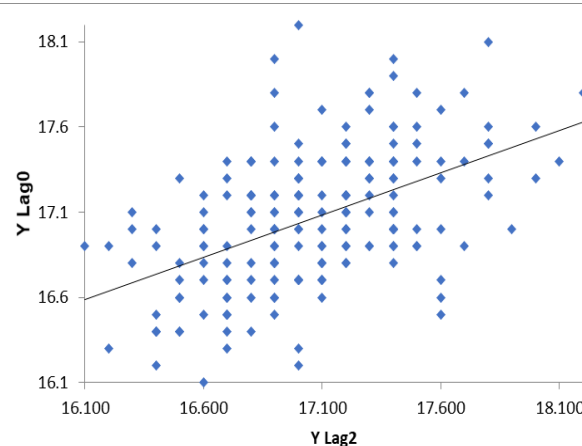
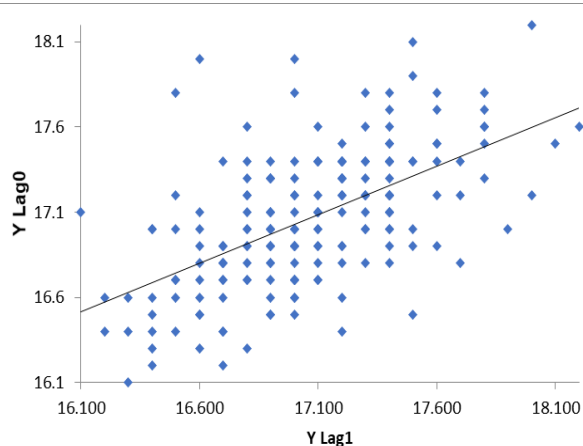


# Autocorrelation

Y Lag0	Y Lag1	Y Lag2	Y Lag3
17			
16.6	17		
16.3	16.6	17	
16.1	16.3	16.6	17
17.1	16.1	16.3	16.6
16.9	17.1	16.1	16.3
16.8	16.9	17.1	16.1
17.4	16.8	16.9	17.1
17.1	17.4	16.8	16.9

Pearson Correlations	Y Lag1	Y Lag2	Y Lag3
Y Lag0	0.571	0.498	0.407

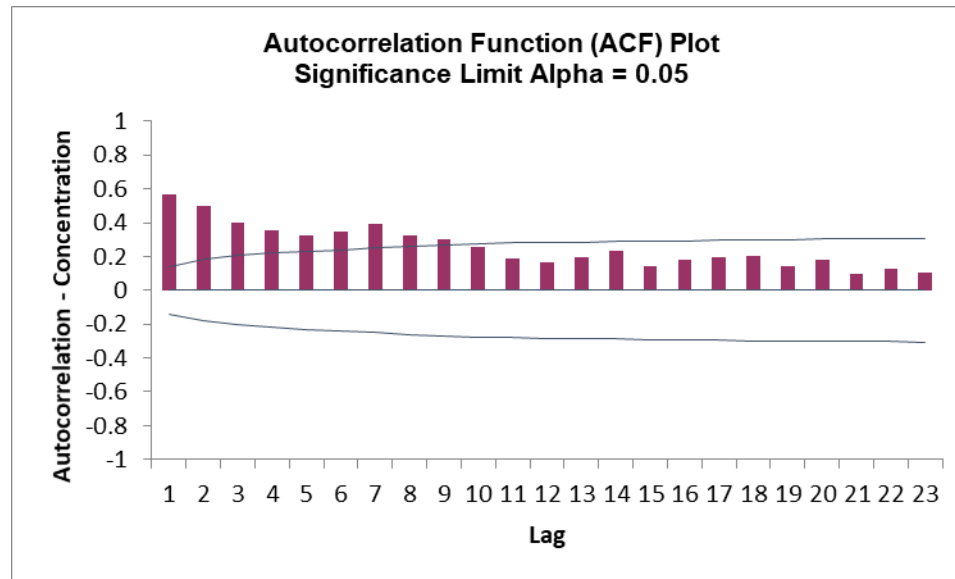
Pearson correlations are used here for demonstration purposes. They are approximately equal to the ACF correlation values.



Any statistically significant correlation ( $r_k > 2/\sqrt{N}$ ) will adversely affect the performance of a Shewhart control chart.

The Ljung-Box test is used to determine if a group of autocorrelations are significant (see formula in Appendix).

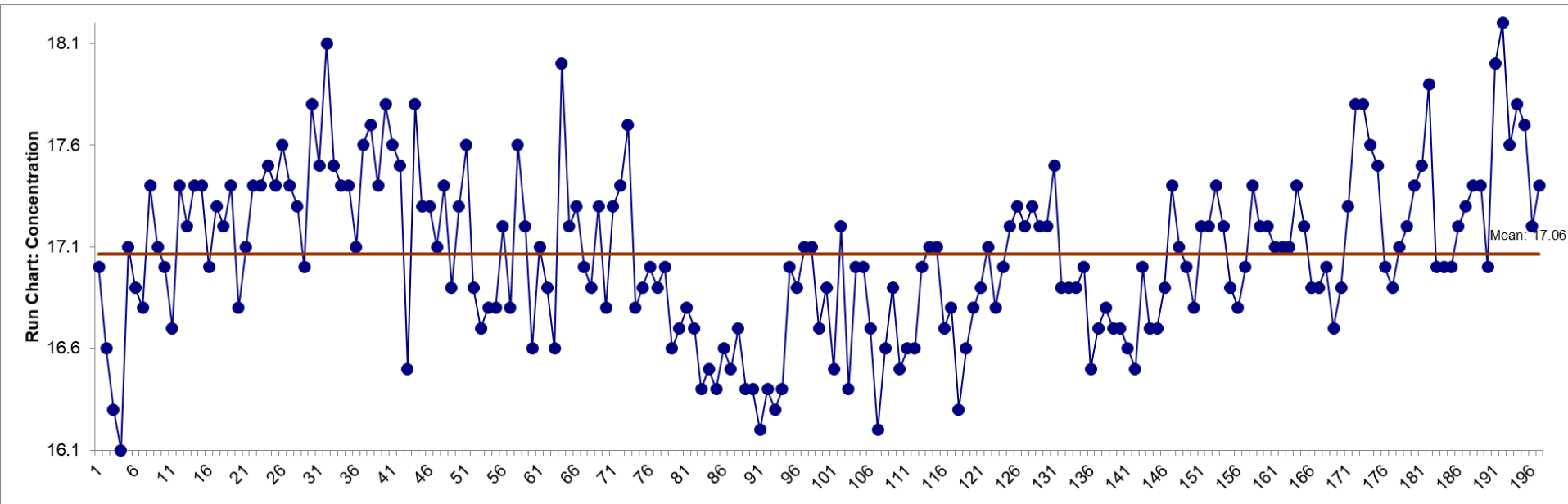
# Example 1: Box-Jenkins Series A - Chemical Process Concentration - Autocorrelation Function (ACF) Plot



SigmaXL > Time Series Forecasting > Autocorrelation (ACF/PACF) Plots  
Example 1: Chemical Process Concentration - Series A.xlsx - Concentration

# Example 1: Box-Jenkins Series A - Chemical Process

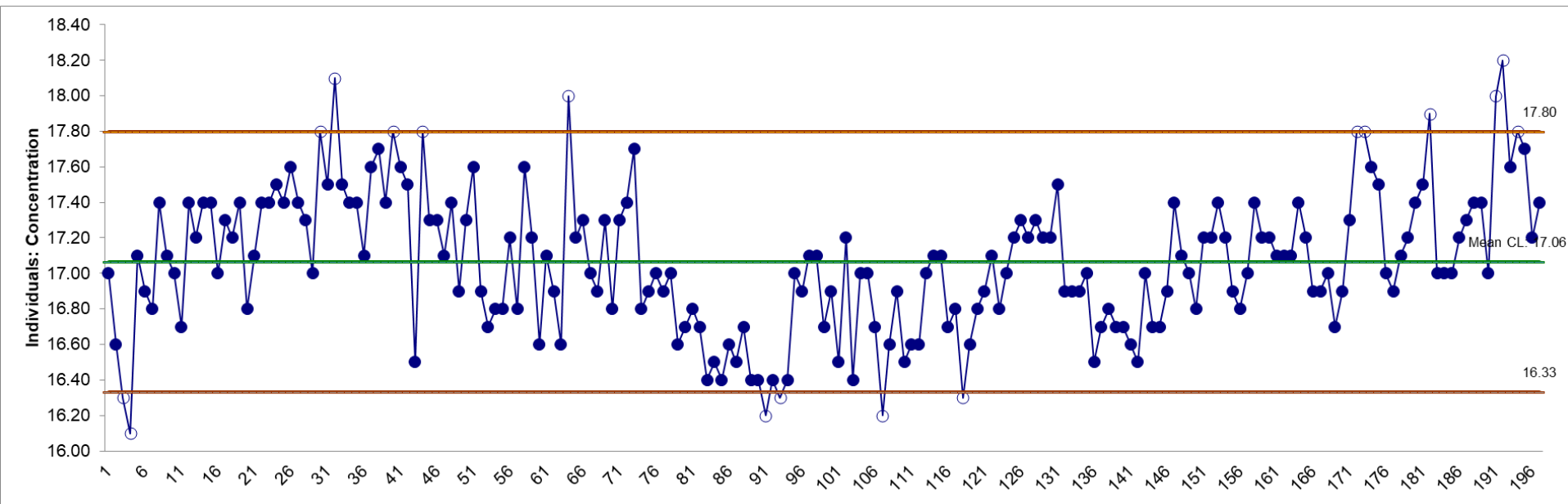
## Concentration - Run Chart



SigmaXL > Time Series Forecasting > Run Chart

# Example 1: Box-Jenkins Series A - Chemical Process

## Concentration - Individuals Control Chart



17 out-of-control data points

SigmaXL > Control Charts > Individuals

# Autocorrelation

Guidelines from Woodall & Faltin [10]:

- If possible, one should first attempt to remove the source of the autocorrelation.
- If the source of autocorrelation cannot be removed directly, then it may be possible to model the autocorrelation and use a feedback control scheme to reduce variability about a specified target value.
- If the source of the autocorrelation cannot be removed directly, and feedback control is not a viable option, then it is important to monitor the process with control charts which do not repeatedly give signals due to presence of the autocorrelation.

# Simple (Single) Exponential Smoothing

## Exponentially Weighted Moving Average (EWMA)

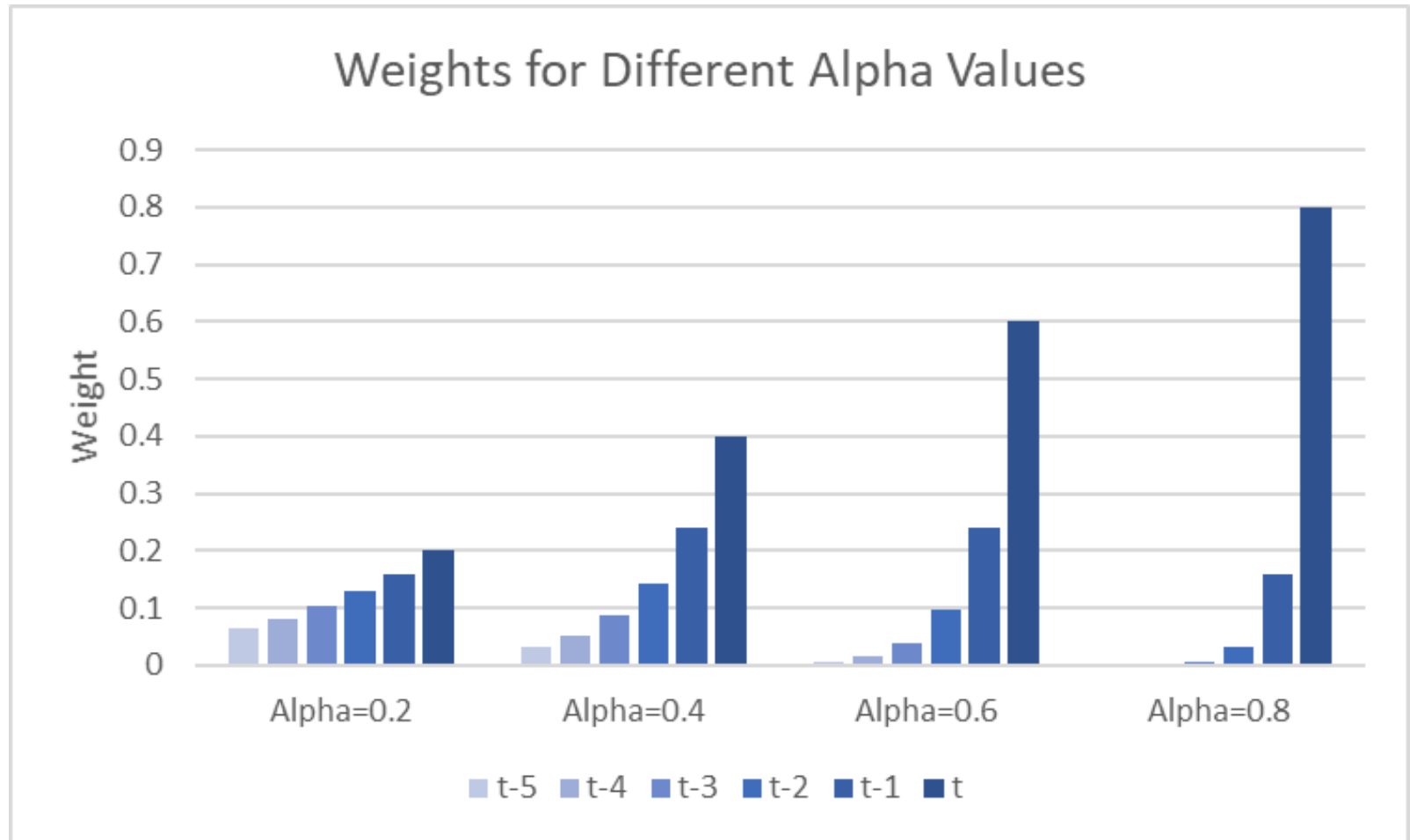
Forecasts are calculated using weighted averages, where the weights decrease exponentially as observations come from further in the past with the smallest weights associated with the oldest observations:

$$\hat{y}_{t+1} = \alpha y_t + \alpha(1 - \alpha) y_{t-1} + \alpha(1 - \alpha)^2 y_{t-2} + \dots$$

where  $0 \leq \alpha \leq 1$  is the level smoothing parameter [4].

# Simple (Single) Exponential Smoothing

## Exponentially Weighted Moving Average (EWMA)



# Simple (Single) Exponential Smoothing

## Exponentially Weighted Moving Average (EWMA)

- An equivalent formulation for simple exponential smoothing is:

$$\hat{y}_{t+1} = \alpha y_t + (1 - \alpha) \hat{y}_t$$

with the starting forecast value (initial level)  $\hat{y}_1$  typically estimated as  $y_1$ .

- The formula used for EWMA is the same, but the smoothing parameter  $\lambda$  is typically used instead of  $\alpha$  and  $X_t$  instead of  $y_t$ :

$$\text{EWMA}_{t+1} = \lambda X_t + (1 - \lambda) \text{EWMA}_t$$

with the starting forecast value  $\text{EWMA}_1$  estimated as the data mean or target value.



# Simple (Single) Exponential Smoothing

## Exponentially Weighted Moving Average (EWMA)

- In the case of an EWMA control chart, the smoothing parameter  $\lambda$  is determined by desired average run length characteristics and is typically 0.2.
- For forecasting or SPC for autocorrelated data, the smoothing parameter and initial level are determined by minimizing the sum-of-square forecast errors (residuals):

$$\text{SSE} = \sum_{t=1}^T (y_t - \hat{y}_t)^2 = \sum_{t=1}^T e_t^2.$$

- This involves non-linear minimization methods like Newton-Raphson or Nelder-Mead Simplex.

# Simple (Single) Exponential Smoothing

## Exponentially Weighted Moving Average (EWMA)

- As usual for any statistical model, the residuals should be normal, independent and identically distributed.
- If this is achieved, this also means that the assumptions for a Shewhart control chart are satisfied.

# Example 1: Box-Jenkins Series A - Chemical Process

## Concentration - Simple Exponential Smoothing (EWMA)

### Time Series Forecast

Exponential Smoothing Forecast

Observation No.

Numeric Time Series Data (Y) >> Concentration

Optional Time Axis Labels >>

<< Remove

OK >> Cancel Help

No. of Forecast Periods 24

Prediction Interval 95.0 %

Specify Model Periods

Start Model at Period 1

Withhold Periods 0

End Model at Period

Model Options

Seasonal Frequency

Specify 12

Select 4 - Quarterly

Automatically Detect

Display ACF/PACF/LB Plots

Display Residual Plots

Box-Cox Transformation

Rounded Lambda

Optimal Lambda

Lambda & Threshold (Shift)

Exponential Smoothing Options

Automatic Model Selection

Specify Model

Error

Additive

Multiplicative

Trend

None

Additive

Additive Damped

Seasonal

None

Additive

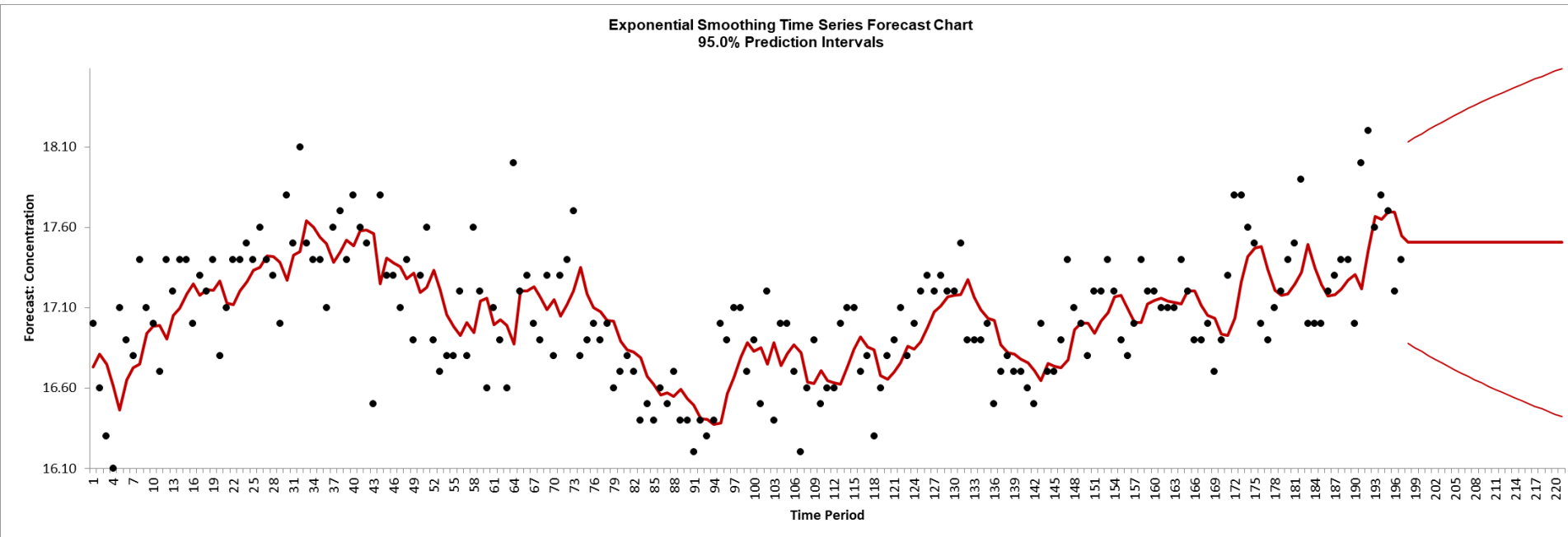
Multiplicative

OK >> Cancel Help

Simple Exponential Smoothing with Additive Errors (A, N, N) - Exponentially Weighted Moving Average (EWMA)

SigmaXL > Time Series Forecasting > Exponential Smoothing Forecast > Forecast

# Example 1: Box-Jenkins Series A - Chemical Process Concentration - Simple Exponential Smoothing (EWMA)



## Exponential Smoothing Model: Concentration

**Model Type:** Simple Exponential Smoothing with Additive Errors (A, N, N) - Exponentially Weighted Moving Average (EWMA)

**Model Periods:** All observations are used in the Exponential Smoothing model estimation. No withhold periods available for out-of-sample forecast accuracy evaluation.

Exponential Smoothing Model Information	
Seasonal Frequency	1
Model selection criterion	AICc
Box-Cox Transformation	N/A
Lambda	
Threshold	

Parameter Estimates	
Term	Coefficient
alpha (level smoothing)	0.294785988
l (level initial state)	16.73121246

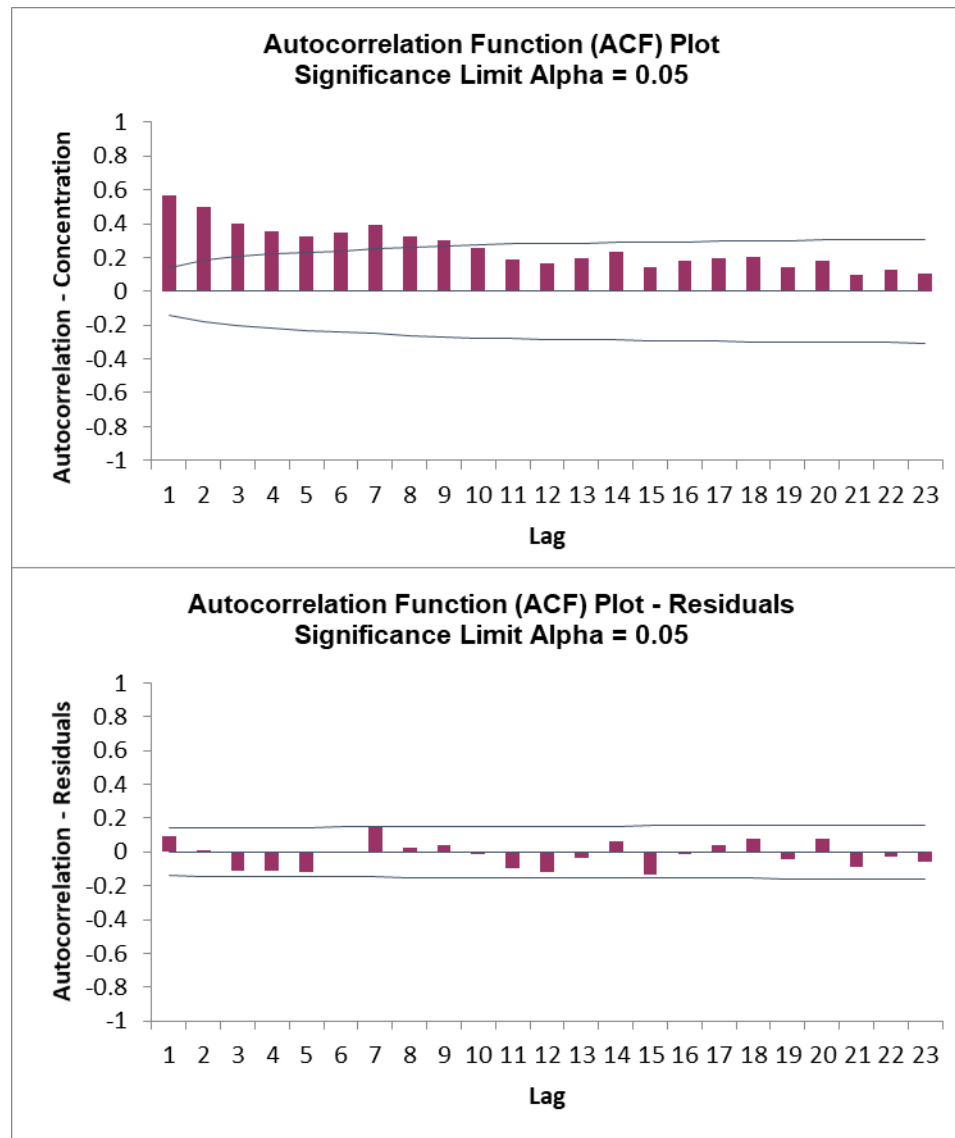
Exponential Smoothing Model Statistics	
No. Observations	197
DF	194
StDev	0.319007644
Variance	0.101765877
Log-Likelihood	-293.8036067
AICc	593.7315658
AIC	593.6072135
BIC	603.4568246

Forecast Accuracy			
Metric	In-Sample (Estimation) One-Step-Ahead Forecast	Out-of-Sample (Withhold) One-Step-Ahead Forecast	Out-of-Sample (Withhold) Full Period Forecast
N	197		
RMSE	0.316569334		
MAE	0.247329038		
MAPE	1.446520183		
MASE	0.897712804		

Simple Exponential Smoothing (EWMA) specified. 95% Prediction Intervals for forecast.

# Example 1: Box-Jenkins Series A - Chemical Process

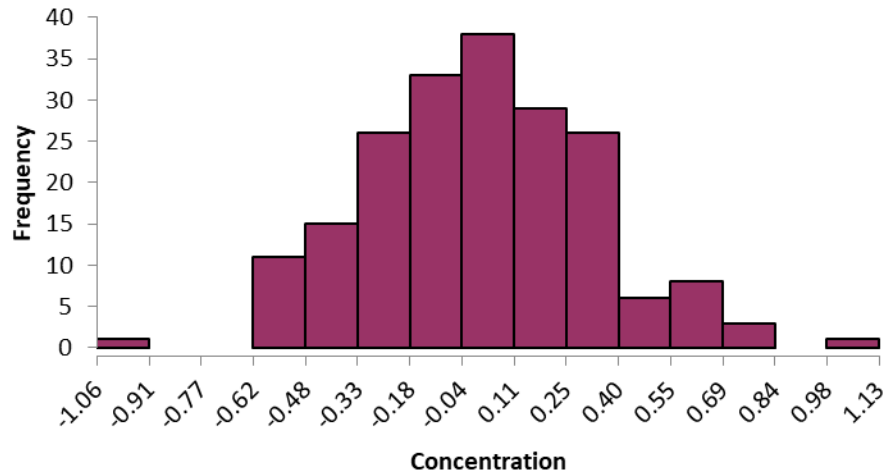
## Concentration - ACF Plots (Raw Data versus Residuals)



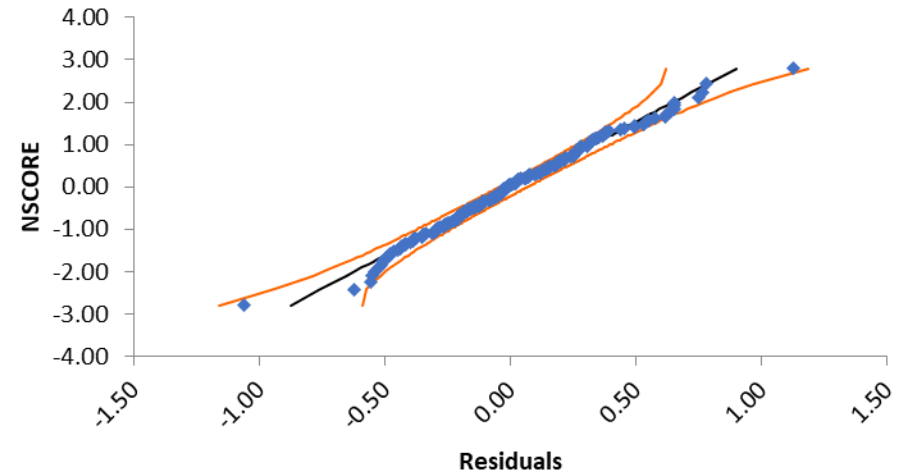
# Example 1: Box-Jenkins Series A - Chemical Process

## Concentration - Residuals

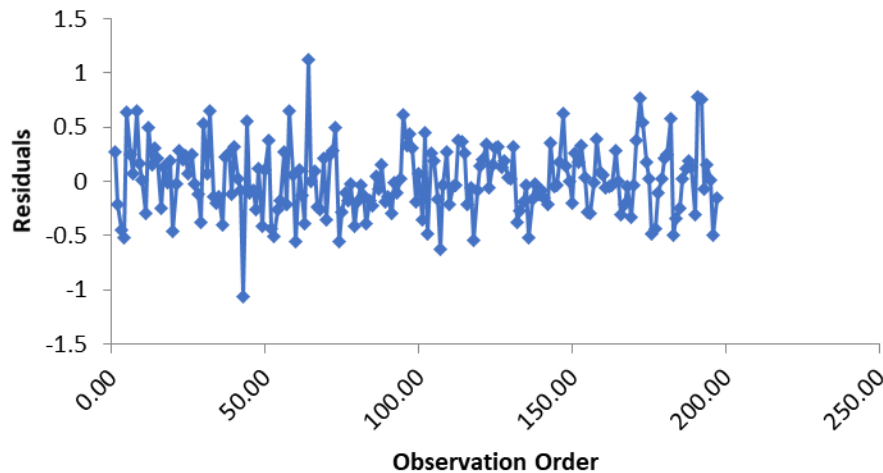
Histogram of Residuals for Concentration



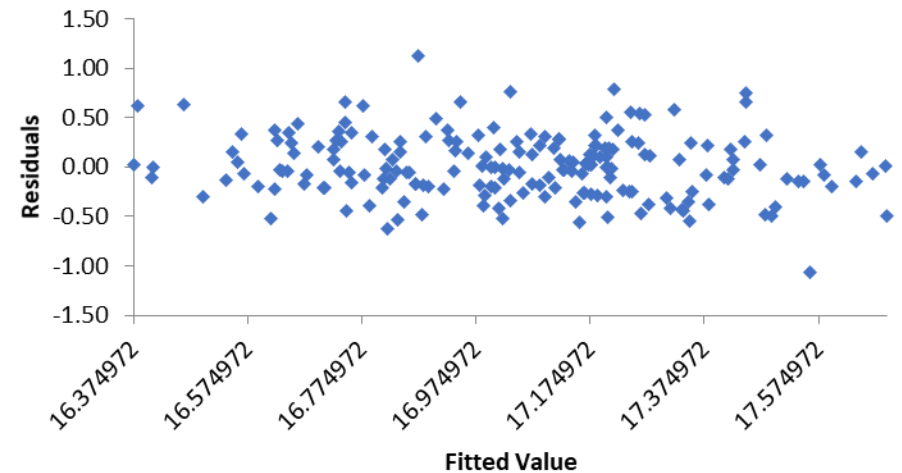
Normal Probability Plot of Residuals for: Concentration



Residuals vs Data Order: Concentration



Residuals vs Forecast Value for: Concentration



Residuals look good – approximately normal with equal variance.

# Example 1: Box-Jenkins Series A - Chemical Process Concentration - Simple Exponential Smoothing (EWMA) Control Chart

The image shows two overlapping dialog boxes from the SigmaXL software. The background dialog is the 'Exponential Smoothing Control Chart' window, and the foreground dialog is the 'Exponential Smoothing Options' window.

**Exponential Smoothing Control Chart (Background):**

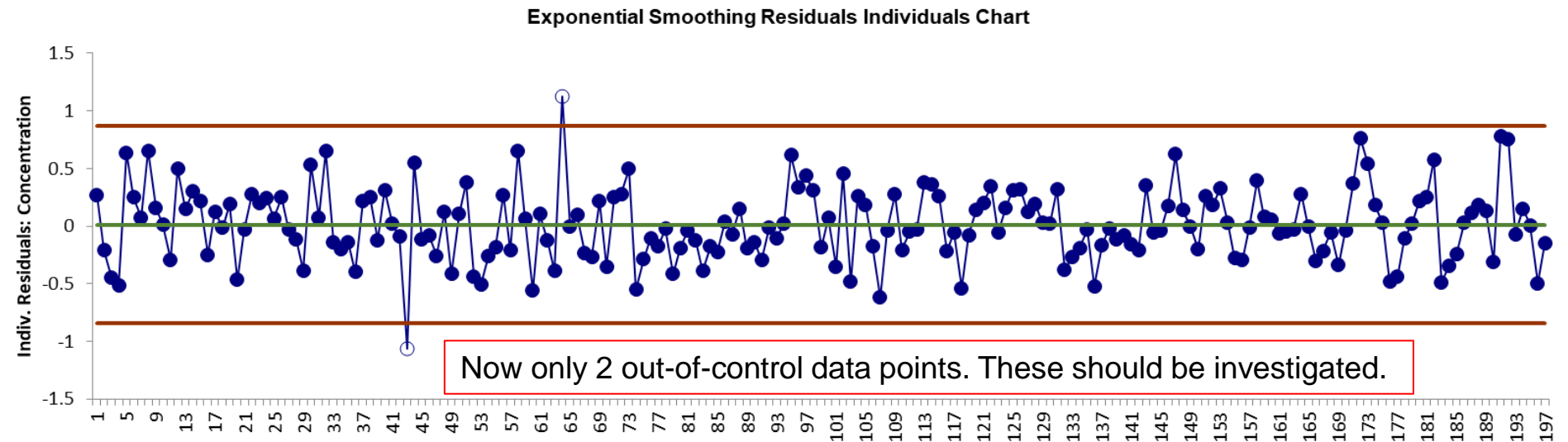
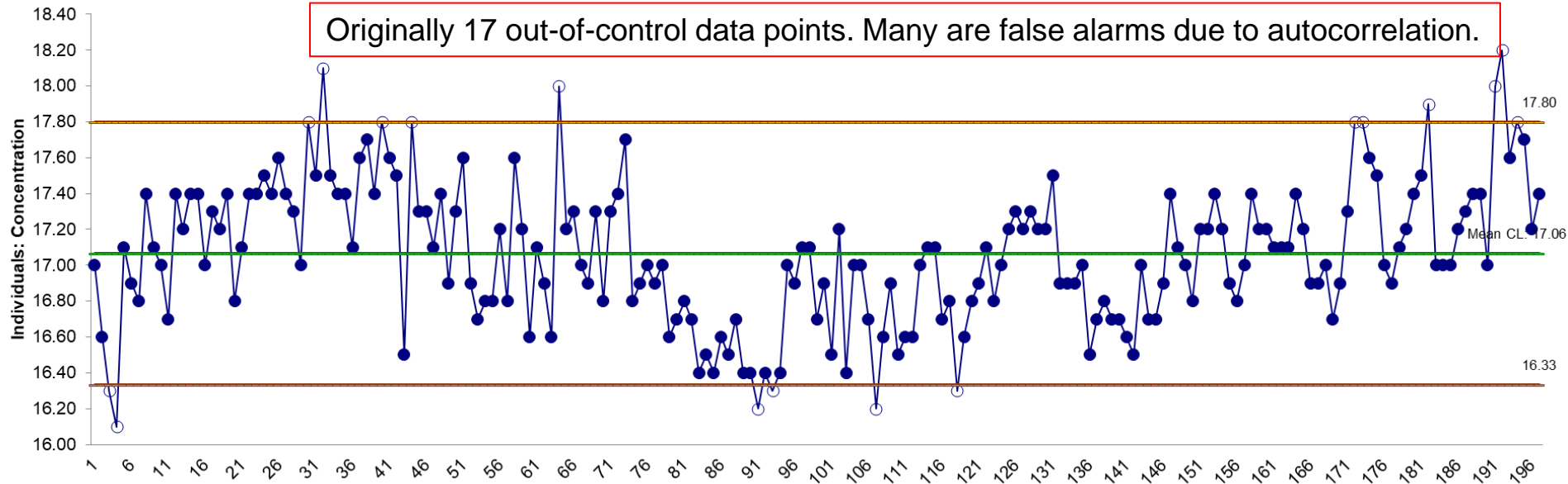
- Observation No.:** A list box for selecting data points.
- Numeric Time Series Data (Y) >>**: A button to select the data series, currently set to 'Concentration'.
- Optional Time Axis Labels >>**: A button to select additional labels.
- << Remove**: A button to remove selected items.
- Model Options**: A section containing:
  - ☐ **Specify Model Periods**: Includes 'Start Model/Control Limit Calculations at Period' (set to 1) and 'End Model/Control Limit Calculations at Period' (empty).
  - ☒ **Withhold Periods**: Set to 0.
  - ☐ **Seasonal Frequency**: Includes 'Specify' (set to 12), 'Select' (set to '4 - Quarterly'), and 'Automatically Detect'.
  - ☐ **Display ACF/PACF/LB Plots**
  - ☐ **Display Residual Plots**
  - ☐ **Box-Cox Transformation**
- Buttons**: 'OK >>', 'Cancel', and 'Help'.

**Exponential Smoothing Options (Foreground):**

- Automatic Model Selection**: ☐ (unselected).
- Specify Model**: ☒ (selected).
- Error**:
  - ☒ **Additive**
  - ☐ **Multiplicative**
- Trend**:
  - ☒ **None**
  - ☐ **Additive**
  - ☐ **Additive Damped**
- Seasonal**:
  - ☒ **None**
  - ☐ **Additive**
  - ☐ **Multiplicative**
- Description**: Simple Exponential Smoothing with Additive Errors (A, N, N) - Exponentially Weighted Moving Average (EWMA).
- Buttons**: 'OK >>', 'Cancel', and 'Help'.

SigmaXL > Time Series Forecasting > Exponential Smoothing Control Chart > Control Chart

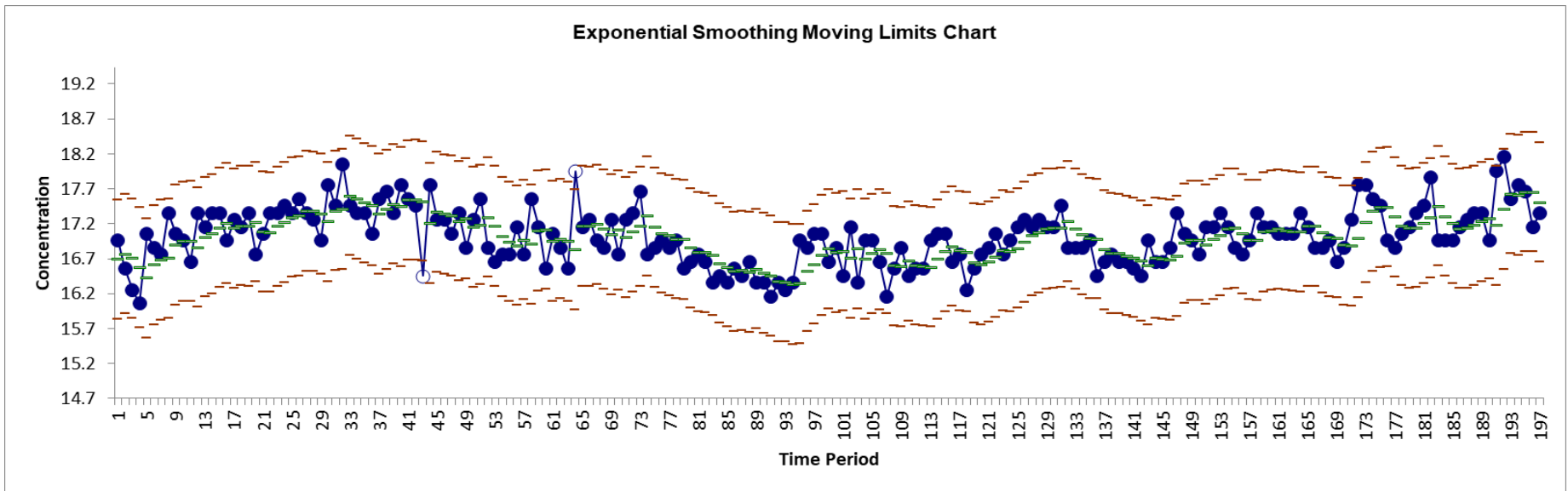
# Example 1: Box-Jenkins Series A - Chemical Process Conc. - Individuals Control Chart (Raw Data versus Residuals)





# Example 1: Box-Jenkins Series A - Chemical Process

## Concentration – Moving Limits Control Chart

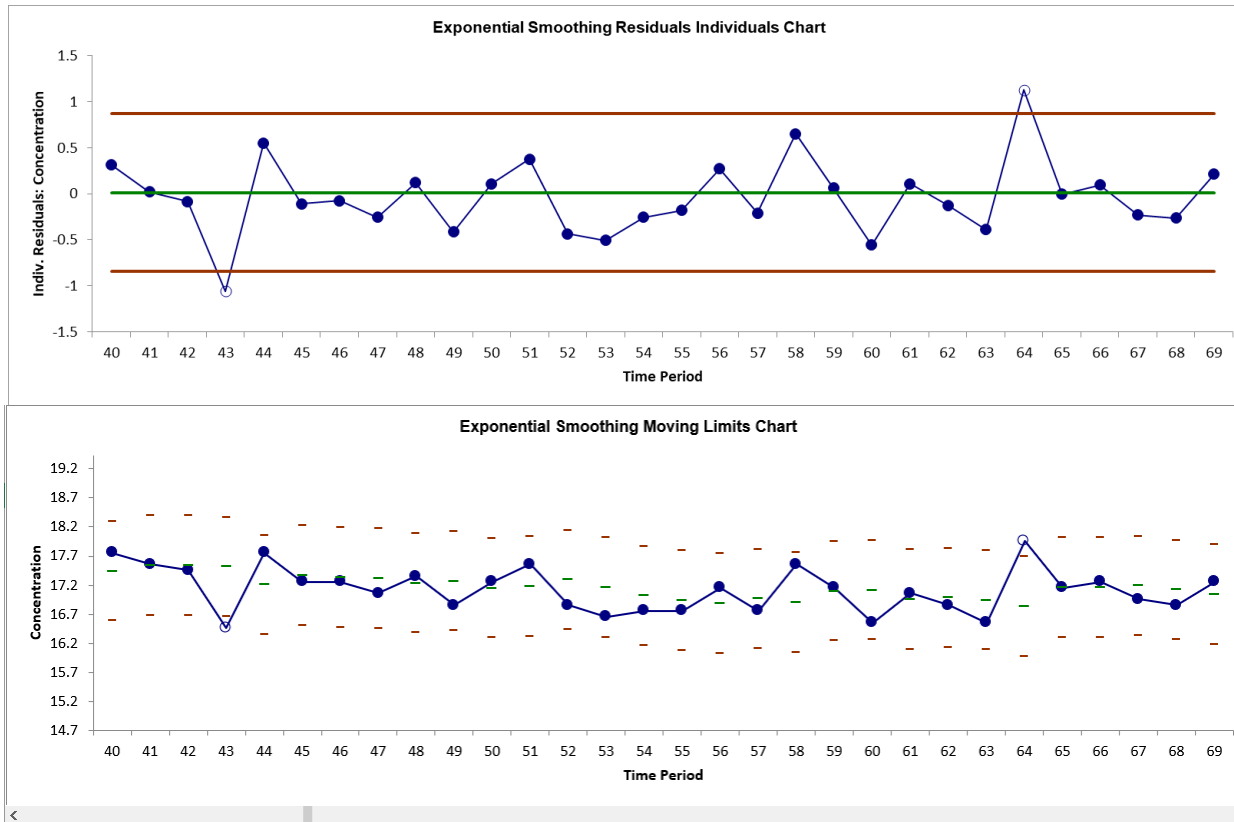


The Moving Limits chart uses the one step prediction as the center line, so the control limits will move with the center line.

SigmaXL > Time Series Forecasting > Exponential Smoothing Control Chart > Control Chart

# Example 1: Box-Jenkins Series A - Chemical Process

## Concentration - Individuals Control Chart: Enable Scrolling



Exponential Smoothing Residuals Individuals Chart

Start Subgroup:

Window Width:

Restore/Show All Data Points

Freeze Chart

Update

Cancel

Help

SigmaXL Chart Tools > Enable Scrolling

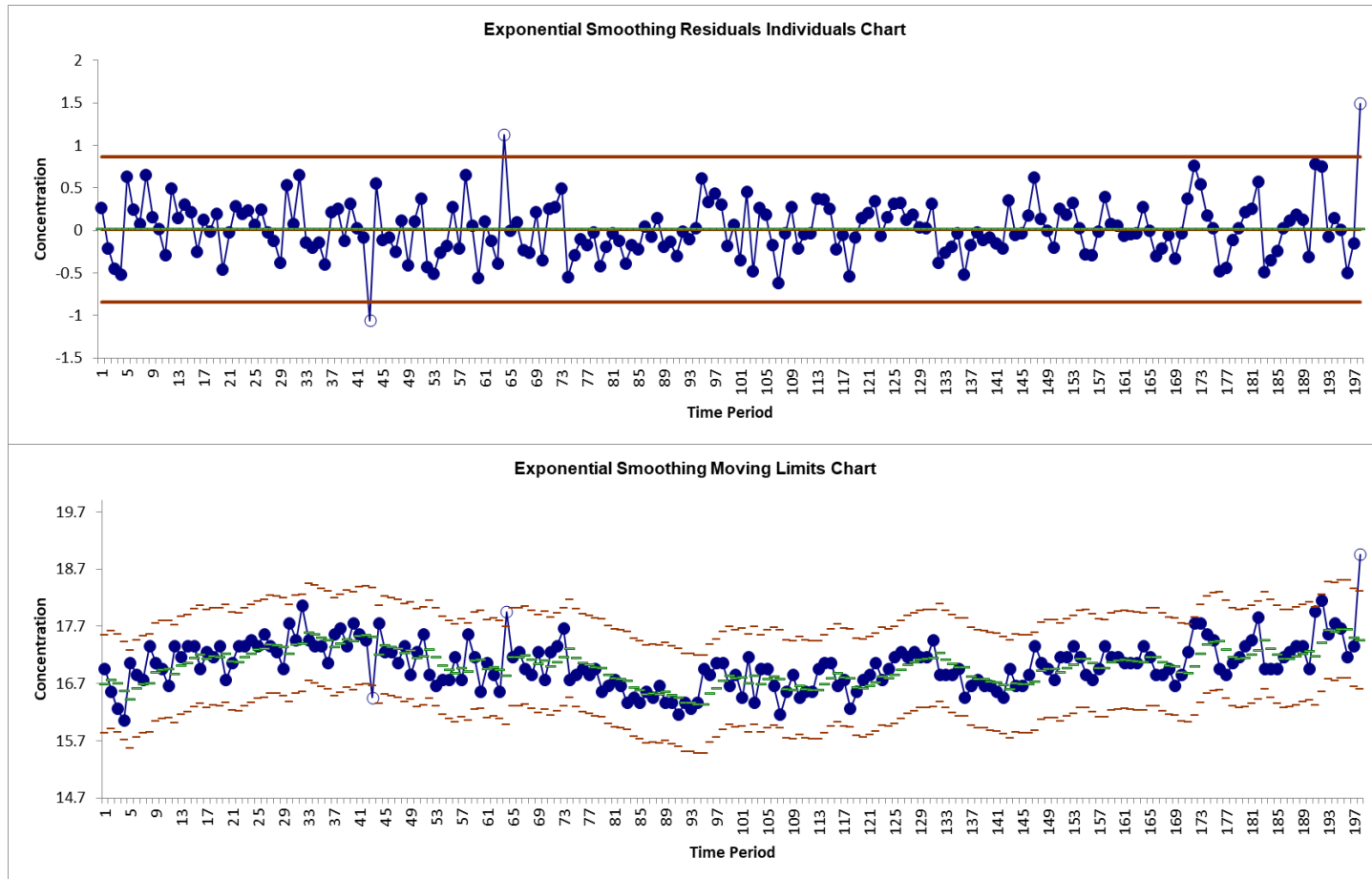
# Example 1: Box-Jenkins Series A - Chemical Process Concentration - Individuals Control Chart: Add Data

Now we will add a new data point to the Series A Concentration Data. The residuals will be computed using the same model as above without re-estimation of the model parameters or recalculation of the control limits. This is also known as the “Phase II” application of a Control Chart, where an out-of-control signal should lead to an investigation into the assignable cause and corrective action or process adjustment applied.

196	195	17.7
197	196	17.2
198	197	17.4
199	198	19

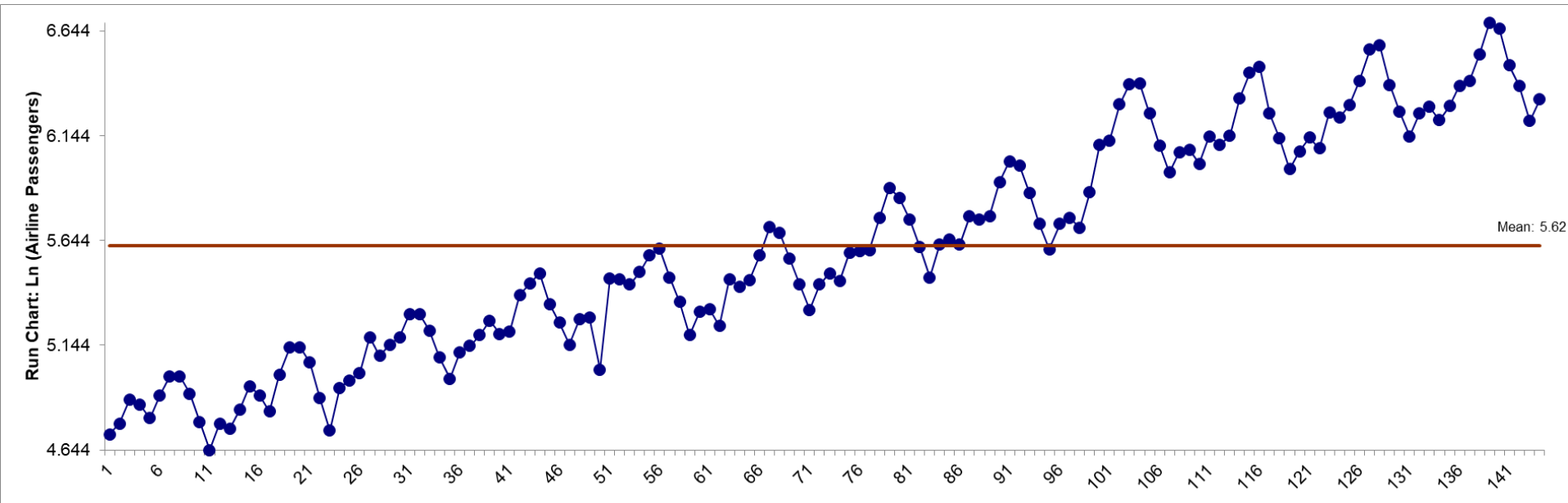
# Example 1: Box-Jenkins Series A - Chemical Process

## Concentration - Individuals Control Chart: Add Data



SigmaXL Chart Tools > Add Data to this Control Chart

## Example 2a: Box-Jenkins Series G – Ln(Monthly Airline Passengers) - Run Chart



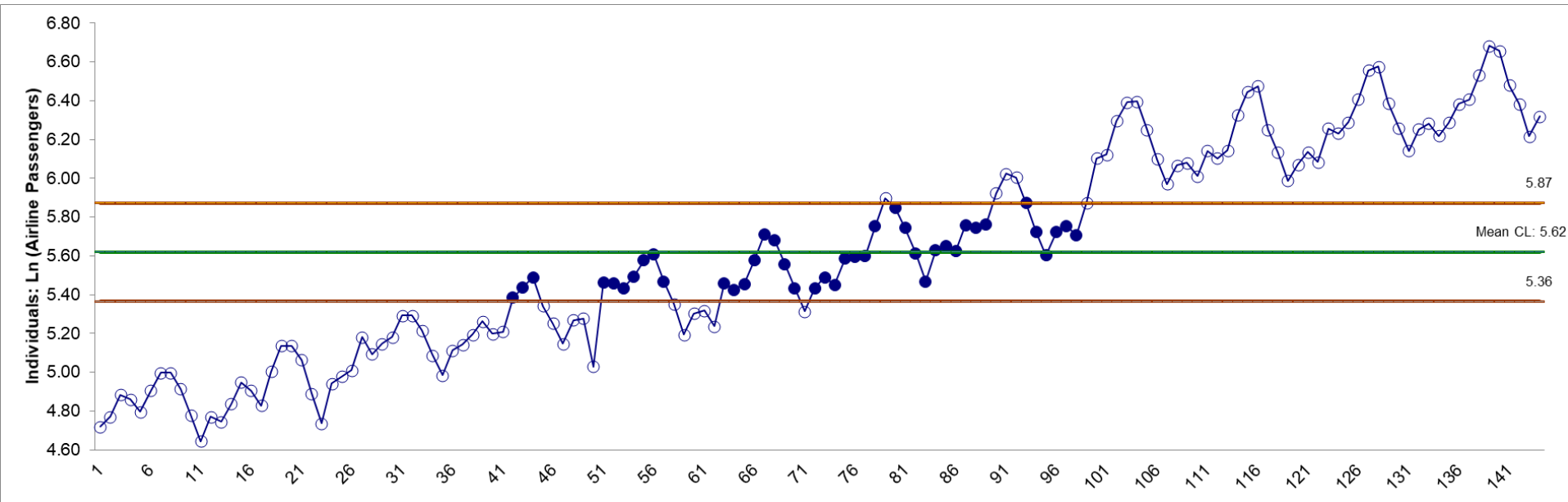
Data modified with negative outlier at 50 (-.25) and level shift (+.25) starting at 100.

Data shows strong positive trend and strong seasonality (monthly data).

SigmaXL > Time Series Forecasting > Run Chart

Example 2: Airline Passengers Modified.xlsx – Ln(Airline Passengers)

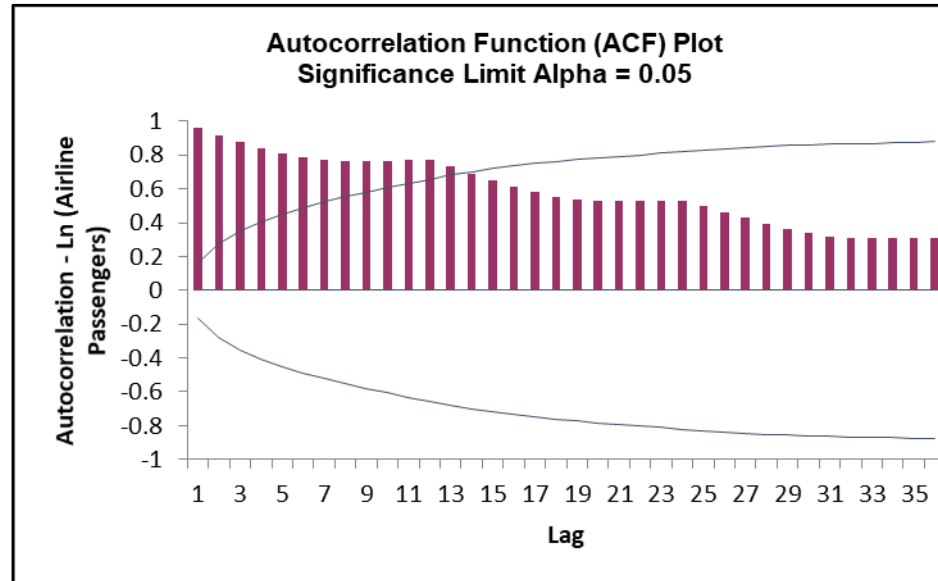
## Example 2a: Box-Jenkins Series G – Ln(Monthly Airline Passengers) - Individuals Control Chart



The control chart signals here are meaningless.

SigmaXL > Control Charts > Individuals

## Example 2a: Box-Jenkins Series G – Ln(Monthly Airline Passengers) - Autocorrelation (ACF) Plot



SigmaXL > Time Series Forecasting > Autocorrelation (ACF/PACF) Plots

# Error, Trend, Seasonal (ETS) Exponential Smoothing Models

- Error, Trend, Seasonal (ETS) models expand on simple exponential smoothing to accommodate trend and seasonal components as well as additive or multiplicative errors.
- Simple Exponential Smoothing is an Error Model.
- Error, Trend model is Holt's Linear, also known as double exponential smoothing.



# Error, Trend, Seasonal (ETS) Exponential Smoothing Models

- Error, Trend, Seasonal model is Holt-Winters, also known as triple exponential smoothing.
  - Seasonal frequency must be specified:
    - Quarterly data = 4 (observations per year)
    - Monthly data = 12 (observations per year)
    - Daily data = 7 (observations per week)
    - Hourly data = 24 (observations per day)
  - Frequency is the number of observations per “cycle”. This is the opposite of the definition of frequency in physics, or in engineering Fourier analysis, where “period” is the length of the cycle, and “frequency” is the inverse of period.

Reference: <https://robjhyndman.com/hyndsight/seasonal-periods/>

# Error, Trend, Seasonal (ETS) models

## Hyndman's Taxonomy

Exponential Model Selection

☐ Automatic Model Selection

☒ Specify Model

Error	Trend	Seasonal
<input checked="" type="radio"/> 1 Additive	<input checked="" type="radio"/> 1 None	<input checked="" type="radio"/> 1 None
<input type="radio"/> 2 Multiplicative	<input type="radio"/> 2 Additive	<input type="radio"/> 2 Additive
	<input type="radio"/> 3 Additive Damped	<input type="radio"/> 3 Multiplicative

Simple Exponential Smoothing with Additive Errors (A, N, N) - Exponentially Weighted Moving Average (EWMA)

OK >>

<< Back

Help

SigmaXL > Time Series Forecasting > Exponential Smoothing Forecast > Forecast

# Example 2b: Box-Jenkins Series G – Ln(Monthly Airline Passengers) – Exponential Smoothing Forecast with Automatic Model Selection

Exponential Smoothing Forecast

Obs. No.  
Outlier 50  
Shift 50  
Outlier 100  
Shift 100

Numeric Time Series Data (Y) >>

Ln (Airline Passengers-Mc

OK >>

Cancel

Help

Optional Time Axis Labels >>

<< Remove

No. of Forecast Periods 24

Prediction Interval 95.0 %

☐ Specify Model Periods

Start Model at Period 1

☒ Withhold Periods 0

☐ End Model at Period

Model Options

☒ Seasonal Frequency

☒ Specify 12

☐ Select 4 - Quarterly

☐ Automatically Detect

☒ Display ACF/PACF/LB Plots

☒ Display Residual Plots

☐ Box-Cox Transformation

☒ Rounded Lambda

☐ Optimal Lambda

☐ Lambda & Threshold (Shift)

Exponential Smoothing Options

☒ Automatic Model Selection

☐ Specify Model

Model Selection Criterion

☒ AICc - Akaike information criterion with small sample size correction

☐ AIC - Akaike information criterion

☐ BIC - Bayesian information criterion

OK >>

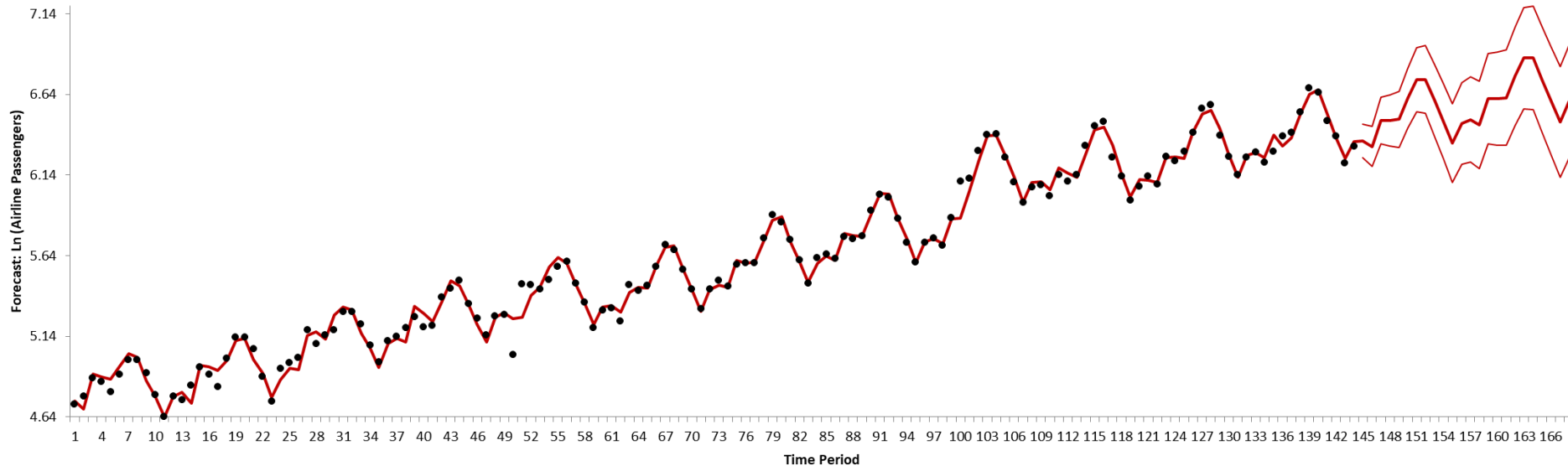
Cancel

Help

SigmaXL > Time Series Forecasting > Exponential Smoothing Forecast > Forecast

# Example 2b: Box-Jenkins Series G – Ln(Monthly Airline Passengers) Seasonal Exponential Smoothing with Trend

Exponential Smoothing Time Series Forecast Chart  
95.0% Prediction Intervals



Exponential Smoothing Model: Ln (Airline Passengers)

Model Type:

Additive Trend, Additive Seasonal Method with Additive Errors (Holt-Winters) (A, A, A)

Model Periods:

All observations are used in the Exponential Smoothing model estimation. No withhold periods available for out-of-sample forecast accuracy evaluation.

Exponential Smoothing Model Information	
Seasonal Frequency	12
Model selection criterion	AICc
Box-Cox Transformation	N/A
Lambda	
Threshold	

Parameter Estimates	
Term	Coefficient
alpha (level smoothing)	0.674949361
beta (trend smoothing)	0.0001
gamma (seasonal smoothing)	0.0001
l (level initial state)	4.821176207
b (trend initial state)	0.01120374
s1 (seasonal initial state)	-0.106060076
s2 (seasonal initial state)	-0.218770169
s3 (seasonal initial state)	-0.073902798
s4 (seasonal initial state)	0.065376172
s5 (seasonal initial state)	0.211146551
s6 (seasonal initial state)	0.221577218
s7 (seasonal initial state)	0.118793494
s8 (seasonal initial state)	-0.00223835
s9 (seasonal initial state)	0.001331568

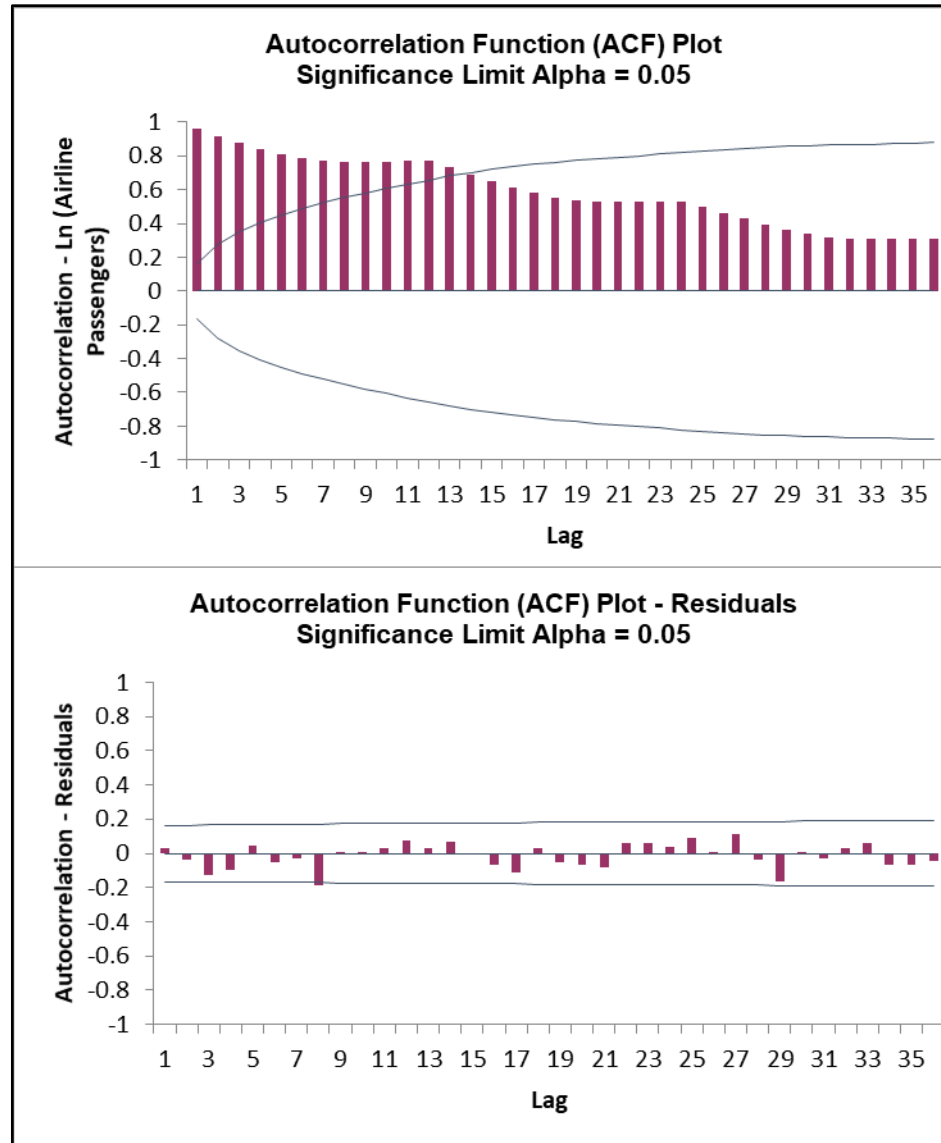
Exponential Smoothing Model Statistics	
No. Observations	144
DF	127
StDev	0.052717869
Variance	0.002779174
Log-Likelihood	74.98184762
AICc	-111.1065524
AIC	-115.9636952
BIC	-65.47686914

Forecast Accuracy			
Metric	In-Sample (Estimation) One-Step-Ahead Forecast	Out-of-Sample (Withhold) One-Step-Ahead Forecast	Out-of-Sample (Withhold) Full Period Forecast
N	144		
RMSE	0.049508351		
MAE	0.033506668		
MAPE	0.609538124		
MASE	0.230072112		

ETS Additive Trend, Additive Seasonal Method with Additive Errors (Holt-Winters) (A, A, A) **automatically selected**. Seasonal Frequency = 12 (Monthly data).

SigmaXL > Time Series Forecasting > Exponential Smoothing Forecast > Forecast

## Example 2b: Box-Jenkins Series G – Ln(Monthly Airline Passengers) - ACF Plots (Raw Data versus Residuals)



# Example 2b: Box-Jenkins Series G – Ln(Monthly Airline Passengers) – Exponential Smoothing Control Chart with Automatic Model Selection

The image shows two dialog boxes from the SigmaXL software. The main dialog is titled "Exponential Smoothing Control Chart" and contains the following elements:

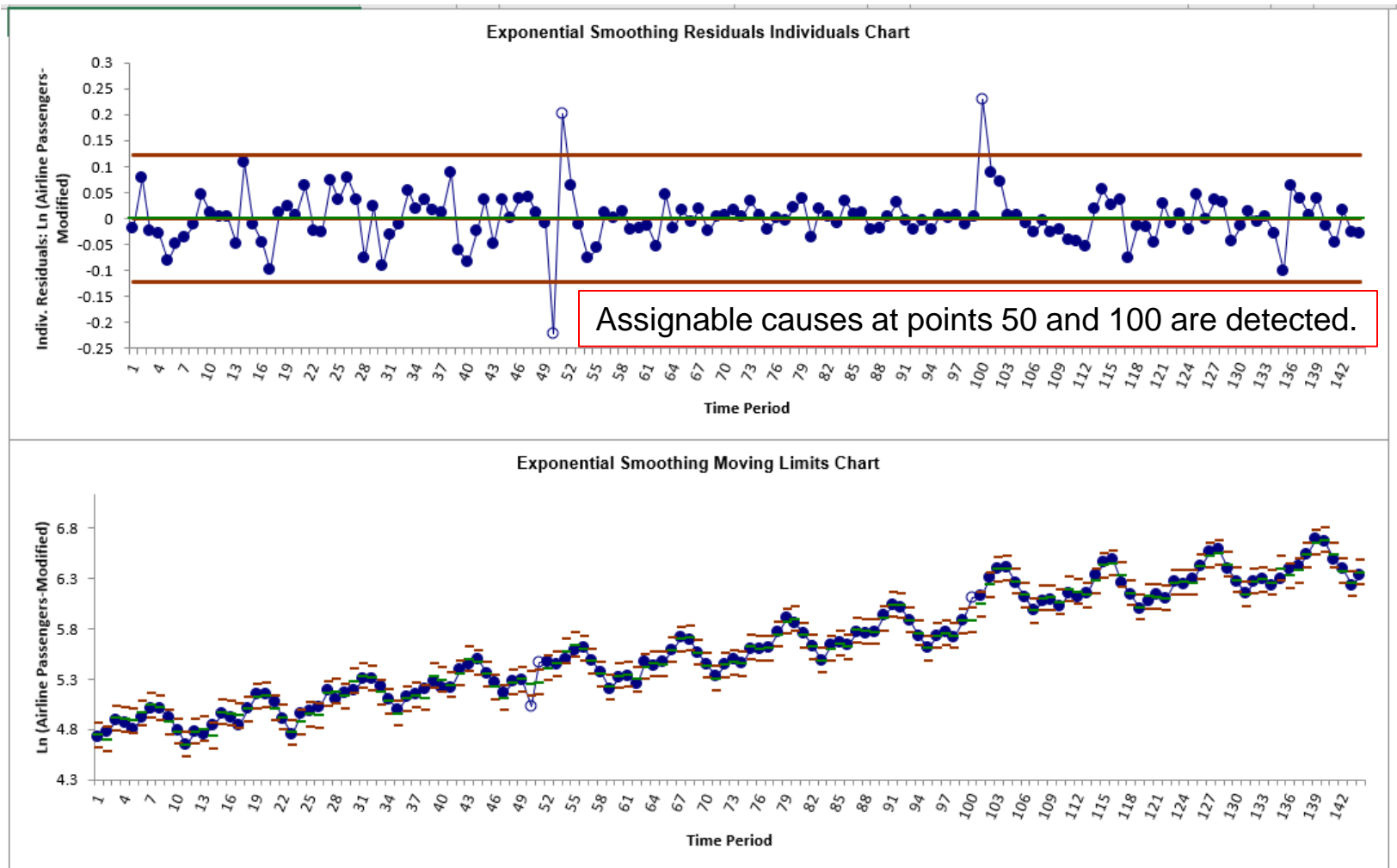
- Obs. No.:** A list with "Outlier 50" selected, followed by "Shift 50", "Outlier 100", and "Shift 100".
- Numeric Time Series Data (Y) >>**: A text box containing "Ln (Airline Passengers-Mc)".
- Optional Time Axis Labels >>**: An empty text box.
- << Remove**: A button.
- Model Options**: A section with several checkboxes and options:
  - ☐ Specify Model Periods
  - Start Model/Control Limit Calculations at Period**: A text box with "1".
  - ☒ Withhold Periods: A text box with "0".
  - ☐ End Model/Control Limit Calculations at Period: A text box.
  - ☒ Seasonal Frequency: A section with:
    - ☒ Specify: A text box with "12".
    - ☐ Select: A dropdown menu showing "4 - Quarterly".
    - ☐ Automatically Detect.
  - ☐ Display ACF/PACF/LB Plots
  - ☐ Display Residual Plots
  - ☐ Box-Cox Transformation
  - ☒ Rounded Lambda
  - ☐ Optimal Lambda
  - ☐ Lambda & Threshold (Shift)
- Buttons:** "OK >>", "Cancel", and "Help".

The second dialog box is titled "Exponential Smoothing Options" and contains the following elements:

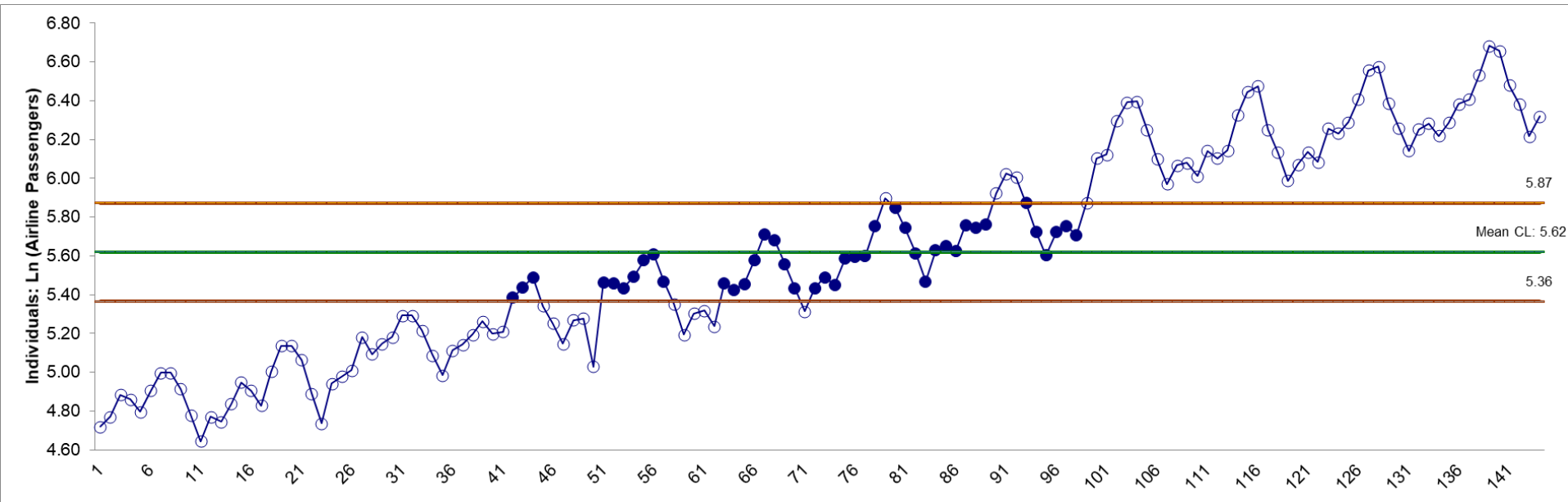
- Automatic Model Selection**: A section with:
  - ☒ Automatic Model Selection
  - ☐ Specify Model
- Model Selection Criterion**: A section with:
  - ☒ AICc - Akaike information criterion with small sample size correction
  - ☐ AIC - Akaike information criterion
  - ☐ BIC - Bayesian information criterion
- Buttons:** "OK >>", "Cancel", and "Help".

SigmaXL > Time Series Forecasting > Exponential Smoothing Control Chart > Control Chart

# Example 2b: Box-Jenkins Series G – Ln(Monthly Airline Passengers) Exponential Smoothing Control Charts



## Example 2b: Box-Jenkins Series G – Ln(Monthly Airline Passengers) - Individuals Control Chart for Raw Data



The control chart signals here are meaningless.

SigmaXL > Control Charts > Individuals



# Box-Jenkins AutoRegressive Integrated Moving Average (ARIMA) Models

- An ARIMA model includes an Autoregressive (AR) component of order  $p$ , an Integrated/Differencing component of order  $d$  and a Moving Average component of order  $q$  and an optional constant.
- An ARIMA Seasonal model includes a Seasonal Autoregressive (SAR) component of order  $P$ , a Seasonal Integrated/Differencing component of order  $D$  and a Seasonal Moving Average component of order  $Q$ .

# Box-Jenkins AutoRegressive Integrated Moving Average (ARIMA) Models

ARIMA Model Selection

☐ Automatic Model Selection  
☒ Specify Model

Nonseasonal Order		Seasonal Order	
AR - Autoregressive (p)	<input type="text" value="0"/>	SAR - Seasonal Autoregressive (P)	<input type="text" value="0"/>
I - Integrated/Differencing (d)	<input type="text" value="1"/>	SI - Seasonal Integrated/Differencing (D)	<input type="text" value="1"/>
MA - Moving Average (q)	<input type="text" value="1"/>	SMA - Seasonal Moving Average (Q)	<input type="text" value="1"/>

☐ Include Constant (Mean if d & D = 0; Trend/Drift if d or D = 1)

OK >>  
<< Back  
Help

SigmaXL > Time Series Forecasting > ARIMA Forecast > Forecast

# Example 2c: Box-Jenkins Series G – Ln(Monthly Airline Passengers) – ARIMA Forecast with Automatic Model Selection

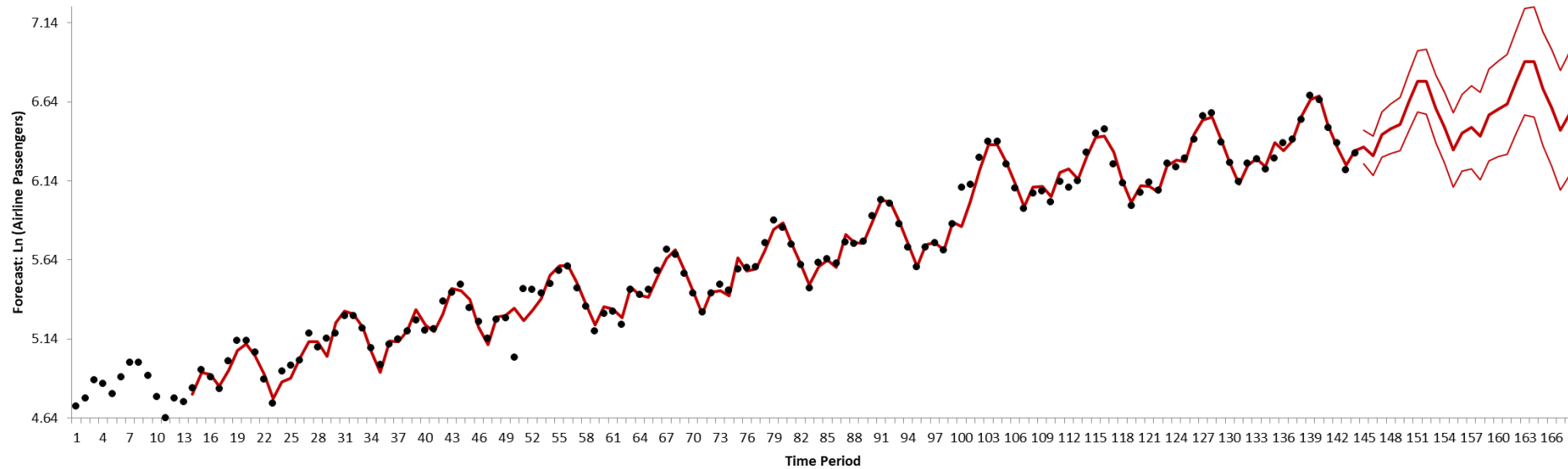
The image shows two dialog boxes from the SigmaXL software. The 'ARIMA Forecast' dialog box on the left is for configuring the forecast. It has a list of observations on the left with 'Outlier 50' selected. The 'Numeric Time Series Data (Y) >>' button is next to 'Ln (Airline Passengers-Mc'. There are buttons for 'OK >>', 'Cancel', 'Help', and '<< Remove'. Below this, there are fields for 'No. of Forecast Periods' (24) and 'Prediction Interval' (95.0 %). A 'Model Options' button is present. The 'Specify Model Periods' section has 'Start Model at Period' (1), 'Withhold Periods' (0), and 'End Model at Period' (empty). The 'Seasonal Frequency' section has 'Specify' (12), 'Select' (4 - Quarterly), and 'Automatically Detect'. The 'Box-Cox Transformation' section has 'Rounded Lambda', 'Optimal Lambda', and 'Lambda & Threshold (Shift)'. The 'Display ACF/PACF/LB Plots' and 'Display Residual Plots' are checked.

The 'ARIMA Model Options' dialog box on the right is for selecting the model. It has 'Automatic Model Selection' selected. The 'Stepwise Procedure' is selected, and the 'Extended Model Search' time limit is 300 seconds. The 'Model Selection Criterion' section has 'AICc - Akaike information criterion with small sample size correction' selected. The 'Specify Nonseasonal Differencing (d)' and 'Specify Seasonal Differencing (D)' are both set to 0.

SigmaXL > Time Series Forecasting > ARIMA Forecast > Forecast

# Example 2c: Box-Jenkins Series G – Ln(Monthly Airline Passengers)

ARIMA Time Series Forecasting Chart  
95.0% Prediction Intervals



ARIMA Model: Ln (Airline Passengers)

Model Periods: All observations are used in the ARIMA model estimation. No withhold periods available for out-of-sample forecast accuracy evaluation.

ARIMA Model Summary	
AR Order (p)	0
I Order (d)	1
MA Order (q)	1
SAR Order (P)	0
SI Order (D)	1
SMA Order (Q)	1
Seasonal Frequency	12
Include Constant	0
No. of Predictors	0
Model selection criterion	AICc
Box-Cox Transformation	N/A
Lambda	

Parameter Estimates				
Term	Coefficient	SE Coefficient	T	P
MA_1	0.387990136	0.091662663	4.2328046	0.0000
SMA_1	0.686391161	0.074905258	9.1634577	0.0000

ARIMA Model Statistics	
No. Observations	144
DF	129
StDev	0.054403004
Variance	0.002959687
Log-Likelihood	191.5831814
AICc	-376.9773864
AIC	-377.1663627
BIC	-368.5407708

Residuals Randomness Runs Test	
P-Value	0.2925

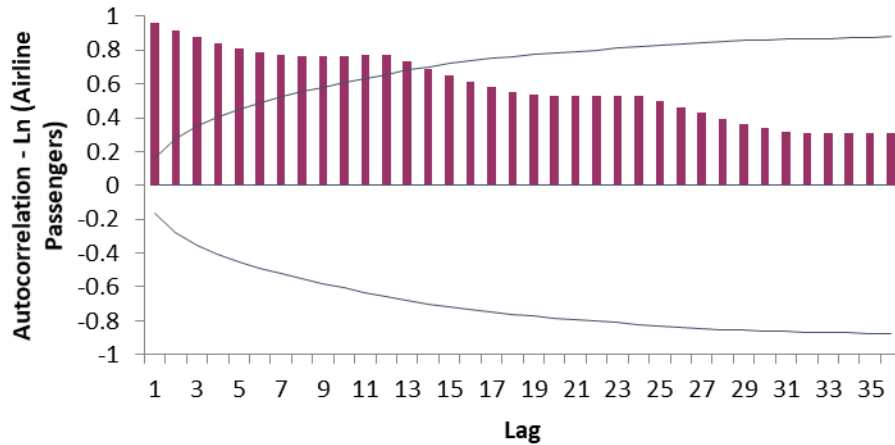
Forecast Accuracy		
Metric	In-Sample (Estimation) One-Step-Ahead Forecast	Out-of-Sample (Withhold) One-Step-Ahead Forecast
N	144	
RMSE	0.055615558	
MAE	0.036285476	
MAPE	0.648152199	
MASE	0.249152676	

ARIMA (0,1,1) (0,1,1) **automatically selected**. Seasonal Frequency = 12 (Monthly data).

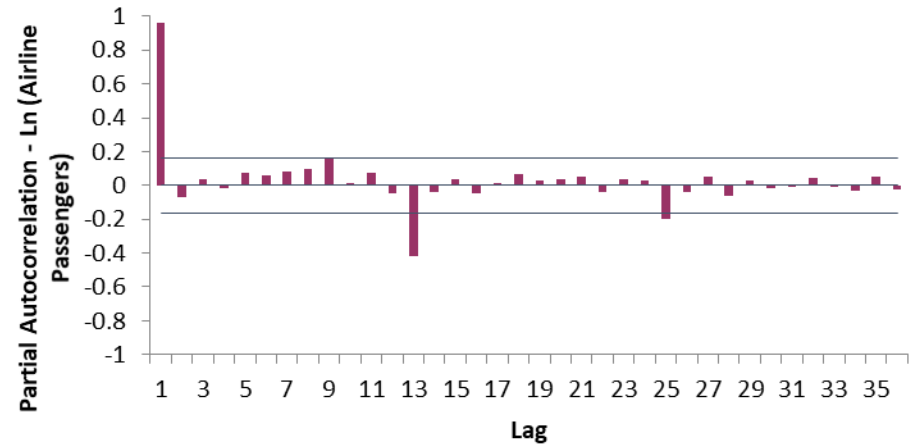
SigmaXL > Time Series Forecasting > ARIMA Forecast > Forecast

# Example 2c: Box-Jenkins Series G – Ln(Monthly Airline Passengers)

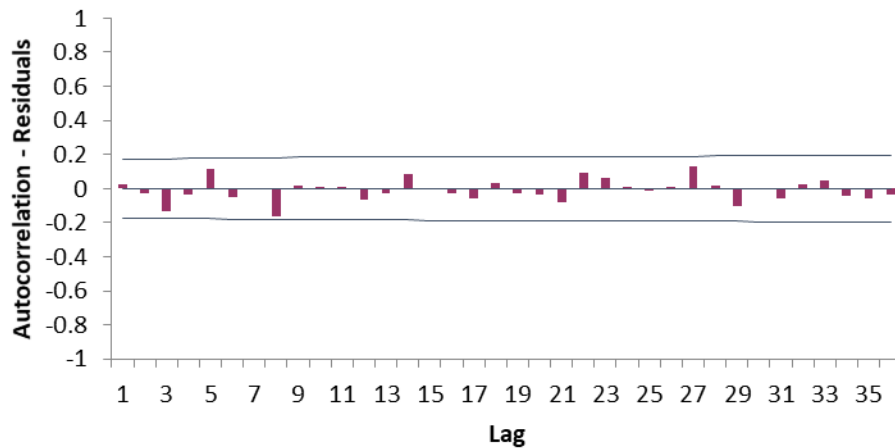
**Autocorrelation Function (ACF) Plot**  
Significance Limit Alpha = 0.05



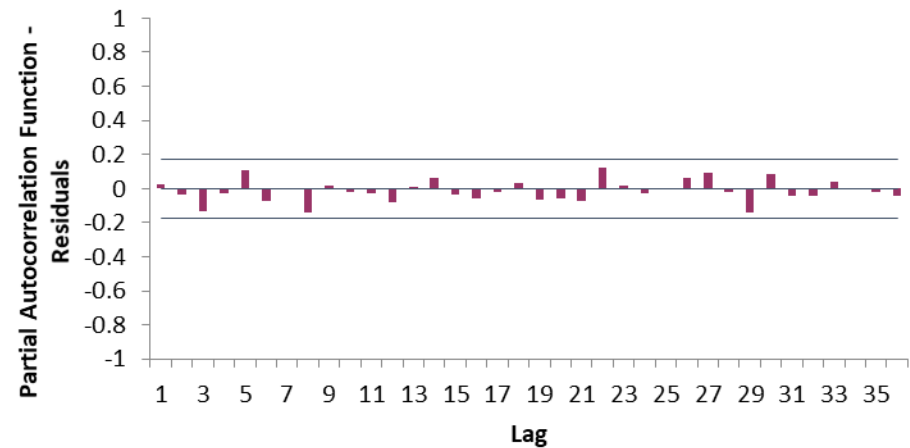
**Partial Autocorrelation Function (PACF) Plot**  
Significance Limit Alpha = 0.05



**Autocorrelation Function (ACF) Plot - Residuals**  
Significance Limit Alpha = 0.05



**Partial Autocorrelation Function (PACF) Plot - Residuals**  
Significance Limit Alpha = 0.05



# Example 2c: Box-Jenkins Series G – Ln(Monthly Airline Passengers) – ARIMA Control Chart with Automatic Model Selection

The image shows two dialog boxes from the SigmaXL software. The 'ARIMA Control Chart' dialog on the left has a list of observations on the left, with 'Outlier 50' selected. It includes buttons for 'Numeric Time Series Data (Y) >>', 'Optional Time Axis Labels >>', '<< Remove', 'OK >>', 'Cancel', and 'Help'. The 'ARIMA Model Options' dialog on the right has tabs for 'Automatic Model Selection' (selected), 'Specify Model', 'Stepwise Procedure', and 'Extended Model Search'. Under 'Automatic Model Selection', it shows 'Model Selection Criterion' with radio buttons for 'AICc - Akaike information criterion with small sample size correction' (selected), 'AIC - Akaike information criterion', and 'BIC - Bayesian information criterion'. It also has checkboxes for 'Specify Nonseasonal Differencing (d)' and 'Specify Seasonal Differencing (D)', both set to 0. Other options include 'Display ACF/PACF/LB Plots', 'Display Residual Plots', 'Box-Cox Transformation', 'Rounded Lambda', 'Optimal Lambda', and 'Lambda & Threshold (Shift)'.

**ARIMA Control Chart**

Obs. No.  
Outlier 50  
Shift 50  
Outlier 100  
Shift 100

Numeric Time Series Data (Y) >> Ln (Airline Passengers-Mc

Optional Time Axis Labels >>

<< Remove

OK >>  
Cancel  
Help

☐ Specify Model Periods

Start Model/Control Limit Calculations at Period 1

☒ Withhold Periods 0

End Model/Control Limit Calculations at Period

☒ Seasonal Frequency

☒ Specify 12

☐ Select 4 - Quarterly

☐ Automatically Detect

☐ Display ACF/PACF/LB Plots

☐ Display Residual Plots

☐ Box-Cox Transformation

☒ Rounded Lambda

☐ Optimal Lambda

☐ Lambda & Threshold (Shift)

**ARIMA Model Options**

☒ Automatic Model Selection

☐ Specify Model

☒ Stepwise Procedure

☐ Extended Model Search. Time limit 300 seconds.

Model Selection Criterion

☒ AICc - Akaike information criterion with small sample size correction

☐ AIC - Akaike information criterion

☐ BIC - Bayesian information criterion

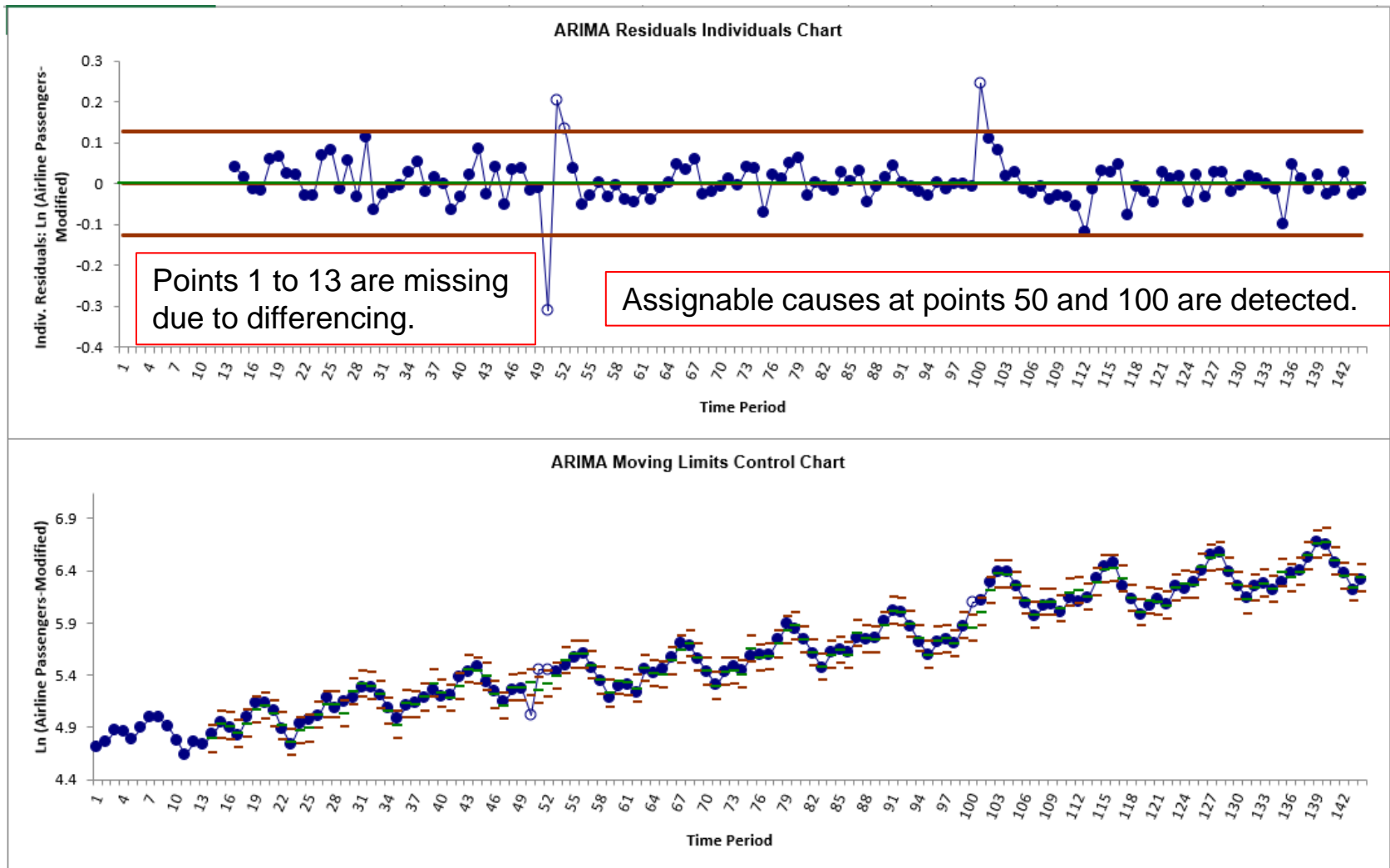
☐ Specify Nonseasonal Differencing (d) 0

☐ Specify Seasonal Differencing (D) 0

OK >>  
Cancel  
Help

SigmaXL > Time Series Forecasting > ARIMA Control Chart > Control Chart

# Example 2c: Box-Jenkins Series G – Ln(Monthly Airline Passengers) ARIMA Control Charts



ARIMA Model: Ln (Airline Passengers-Modified) - Model Automatically Selected

Model Periods: All observations are used in the ARIMA model estimation. No withhold periods available for out-of-sample forecast accuracy evaluation.

# ARIMA with Predictors

- The ARIMA model supports continuous or categorical predictors, similar to multiple regression.
- In order to provide a forecast, additional predictor (X) values must be added to the dataset prior to running the analysis. The number of forecast periods will be equal to the number of additional predictor rows. Alternatively, the predictor values from a withhold sample may be used.
- As with multiple linear regression, predictors should not be strongly correlated.



## Example 2d: Box-Jenkins Series G – Ln(Monthly Airline Passengers) – Outlier versus Shift Coded Predictors

Obs. No.	Ln (Airline Passengers-Modified)	Outlier 50	Shift 50	Outlier 100	Shift 100
49	5.278114659	0	0	0	0
50	5.028114659	1	1	0	0
51	5.463831805	0	1	0	0
52	5.459585514	0	1	0	0
53	5.433722004	0	1	0	0
54	5.493061443	0	1	0	0
55	5.575949103	0	1	0	0
99	5.874930731	0	1	0	0
100	6.10220248	0	1	1	1
101	6.122117789	0	1	0	1
102	6.295005314	0	1	0	1
103	6.392037406	0	1	0	1
104	6.396329258	0	1	0	1
105	6.251414878	0	1	0	1

# Example 2d: Box-Jenkins Series G – Ln(Monthly Airline Passengers) – ARIMA Forecast with Predictors: Outlier versus Shift Coded Predictors

ARIMA with Predictors Forecast

Obs. No.

Numeric Time Series Data (Y) >> Ln (Airline Passengers-Mc

Optional Time Axis Labels >>

Optional Continuous Pred. (X) >> Outlier 50  
Shift 50  
Outlier 100

Optional Categorical Pred. (X) >>

<< Remove

Model Options

No. of Forecast Periods 24

Prediction Interval 95.0 %

Specify Model Periods

Start Model at Period 1

Withhold Periods 0

End Model at Period

Seasonal Frequency

Specify 12

Select 4-Quarterly

Automatically Detect

Display ACF/PACF/LB Plots

Display Residual Plots

Box-Cox Transformation

Rounded Lambda

Optimal Lambda

Lambda & Threshold (Shift)

ARIMA Model Options

Automatic Model Selection

Specify Model

Stepwise Procedure

Extended Model Search. Time limit 300 seconds.

Model Selection Criterion

AICc - Akaike information criterion with small sample size correction

AIC - Akaike information criterion

BIC - Bayesian information criterion

Specify Nonseasonal Differencing (d) 0

Specify Seasonal Differencing (D) 0

SigmaXL > Time Series Forecasting > ARIMA Forecast > Forecast with Predictors

## Example 2d: Box-Jenkins Series G – Ln(Monthly Airline Passengers) – Outlier versus Shift Coded Predictors

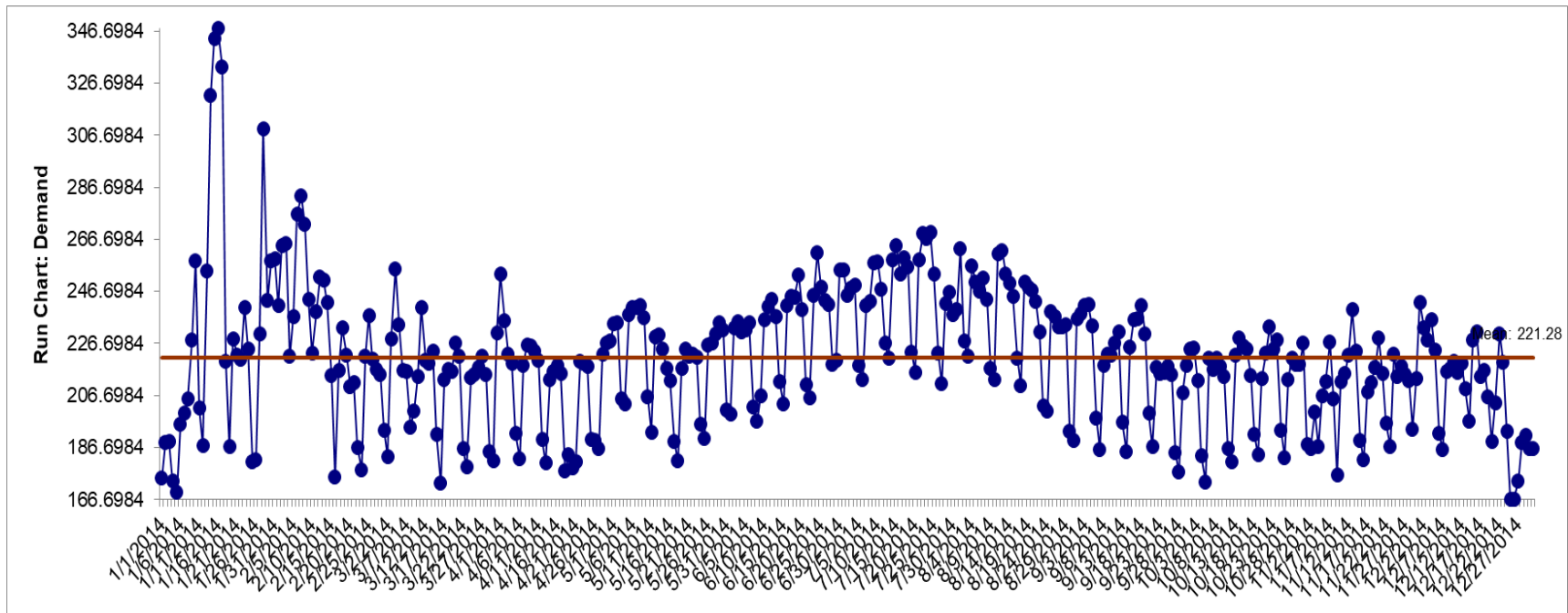
Using ARIMA Forecast with Predictors, we can see that *Outlier50* and *Shift100* are significant denoting Obs. No. 50 as an outlier and 100 as a shift. This is, of course, what we expected since that's how the Ln Airline Passenger data was modified.

This method to identify outlier versus shift is intended as a complement to process knowledge and the search for assignable causes used in classical SPC.

Parameter Estimates				
Term	Coefficient	SE Coefficient	T	P
MA_1	0.395854258	0.086277876	4.588132	0.0000
SMA_1	0.556926317	0.073521579	7.575005	0.0000
Outlier 50	-0.296267312	0.032292885	9.174384	0.0000
Shift 50	0.044130277	0.035505653	1.242908	0.2162
Outlier 100	-0.000597974	0.031822469	0.018791	0.9850
Shift 100	0.249993132	0.035384072	7.065132	0.0000

SigmaXL > Time Series Forecasting > ARIMA Forecast > Forecast with Predictors

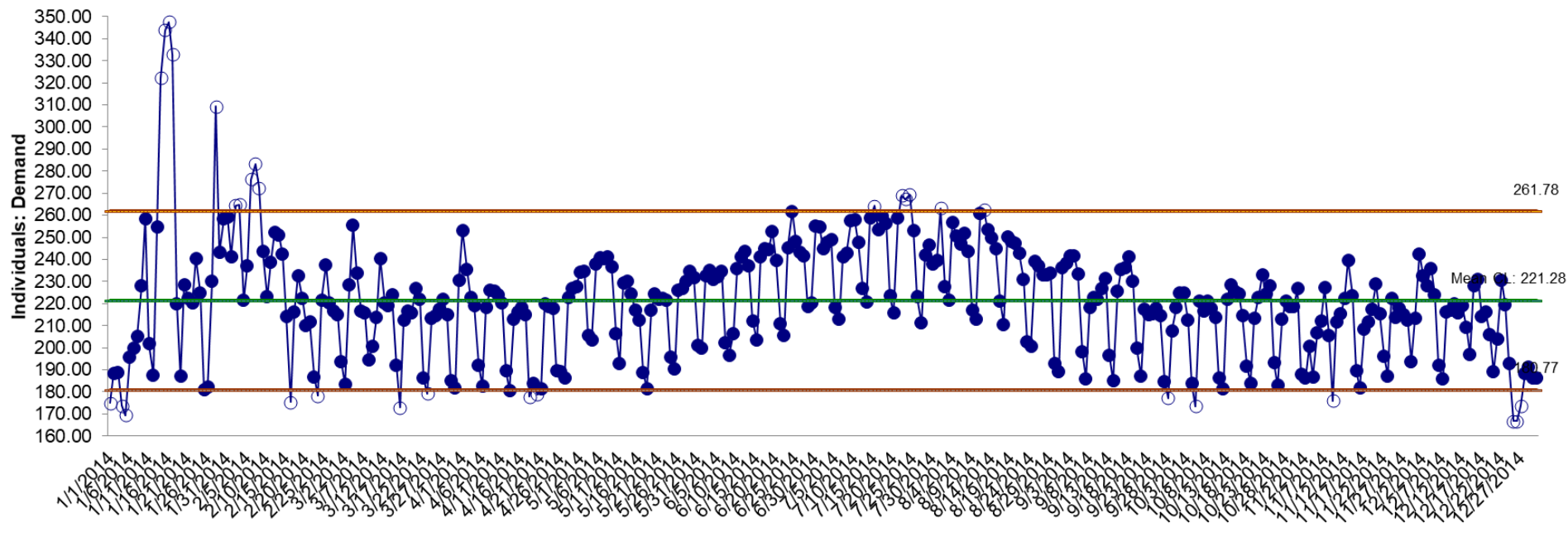
# Example 3: Daily Electricity Demand with Temperature and Work Day Predictors – Run Chart



SigmaXL > Time Series Forecasting > Run Chart

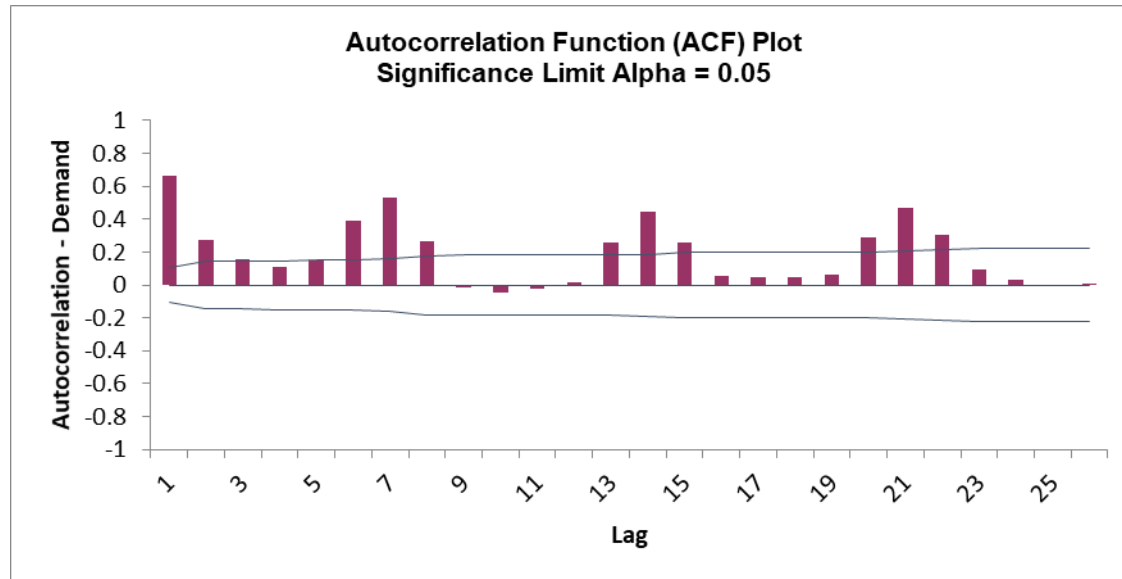
Example 3: Daily Electricity Demand with Predictors – ElecDaily.xlsx  
Victoria, Australia, 2014.

## Example 3: Daily Electricity Demand with Temperature and Work Day Predictors - Individuals Control Chart



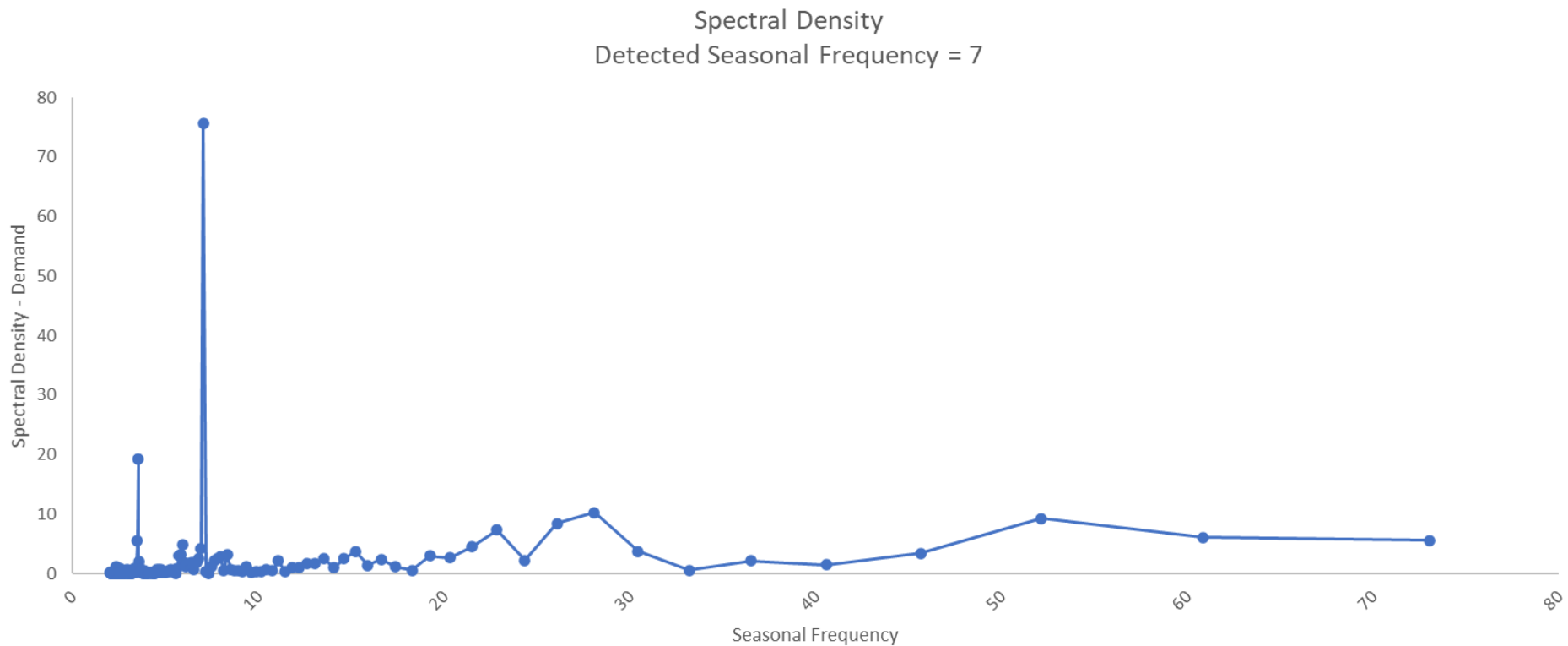
SigmaXL > Control Charts > Individuals

## Example 3: Daily Electricity Demand with Temperature and Work Day Predictors – Autocorrelation (ACF) Plot



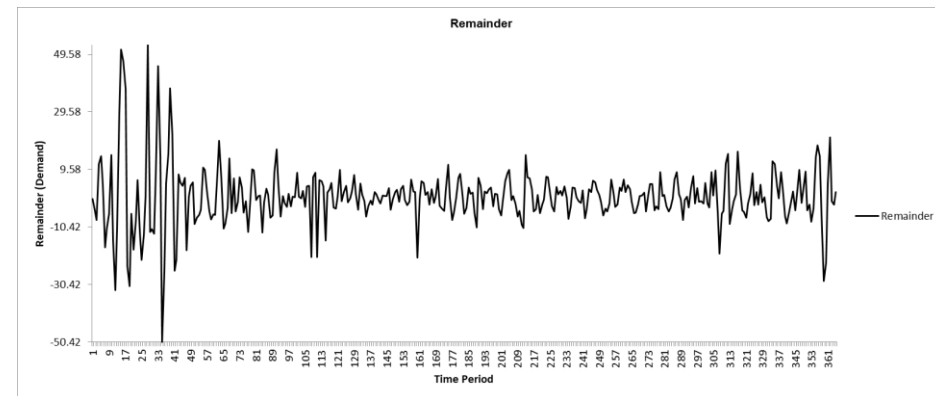
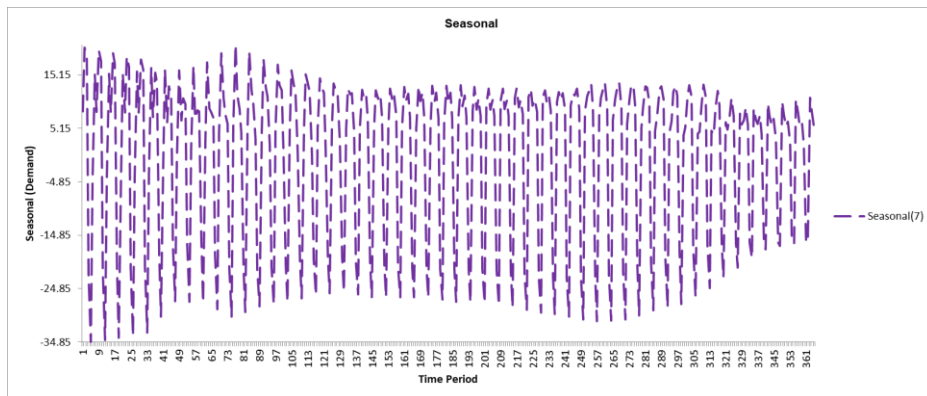
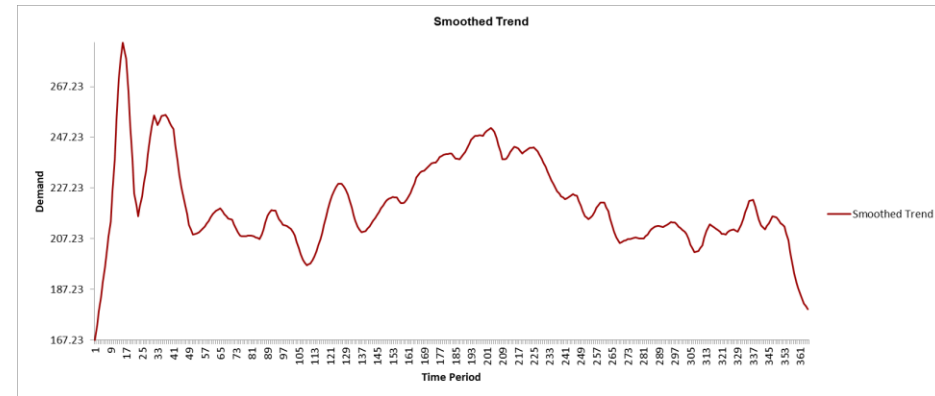
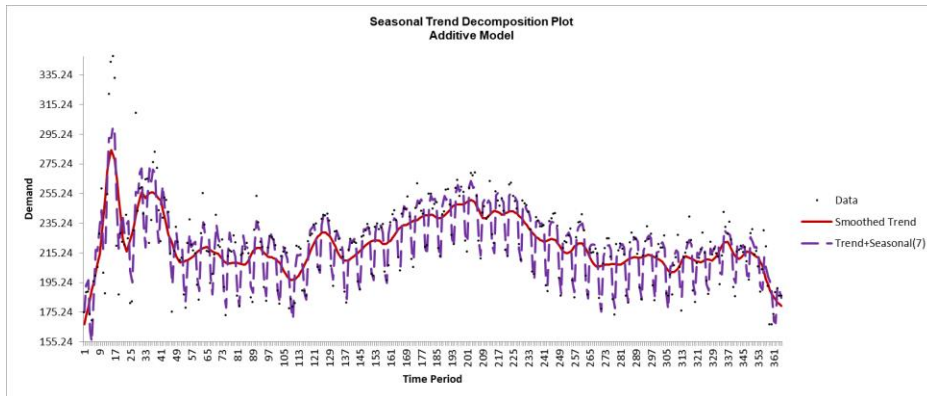
SigmaXL > Time Series Forecasting > Autocorrelation (ACF/PACF) Plots

## Example 3: Daily Electricity Demand with Temperature and Work Day Predictors - Spectral Density Plot



SigmaXL > Time Series Forecasting > Spectral Density Plot

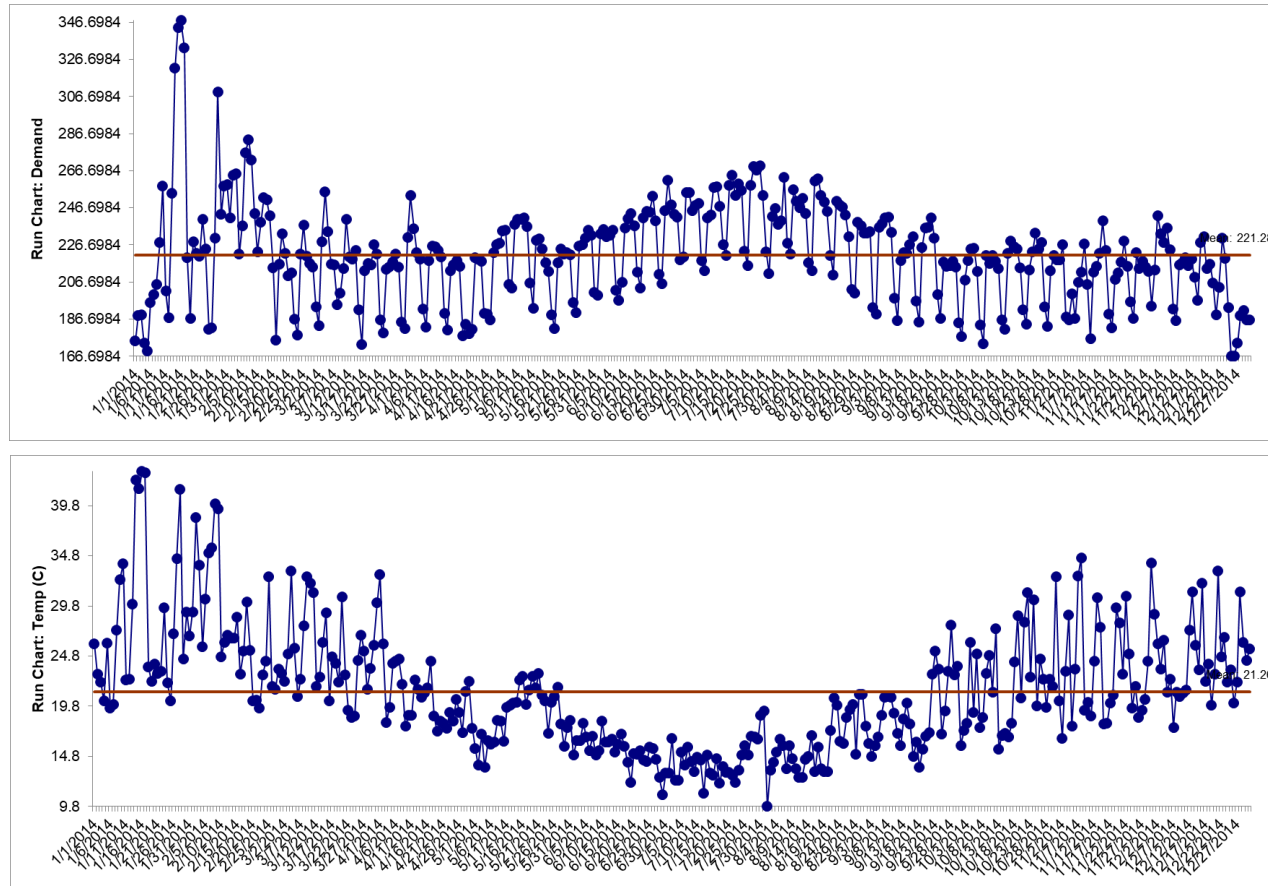
# Example 3: Daily Electricity Demand with Temperature and Work Day Predictors - Seasonal Trend Decomposition



SigmaXL > Time Series Forecasting > Seasonal Trend Decomposition Plots



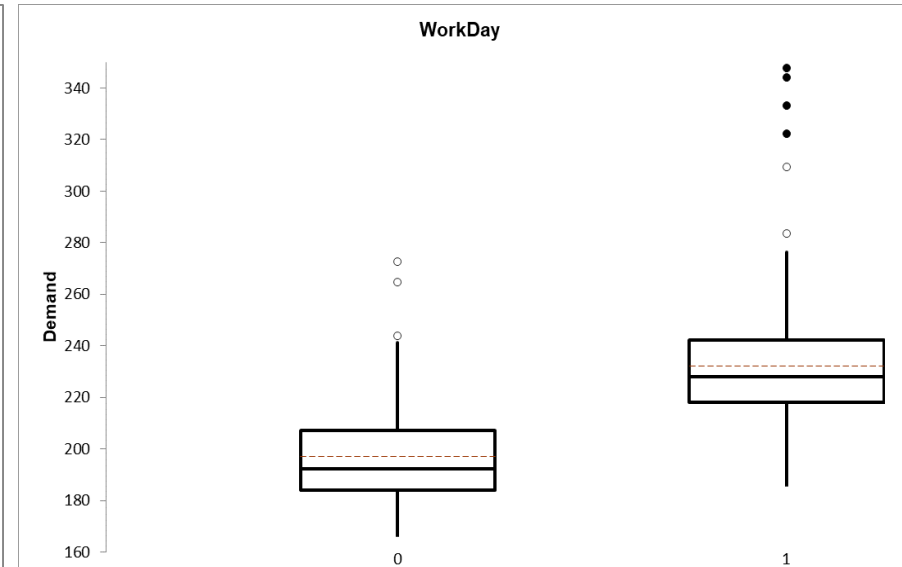
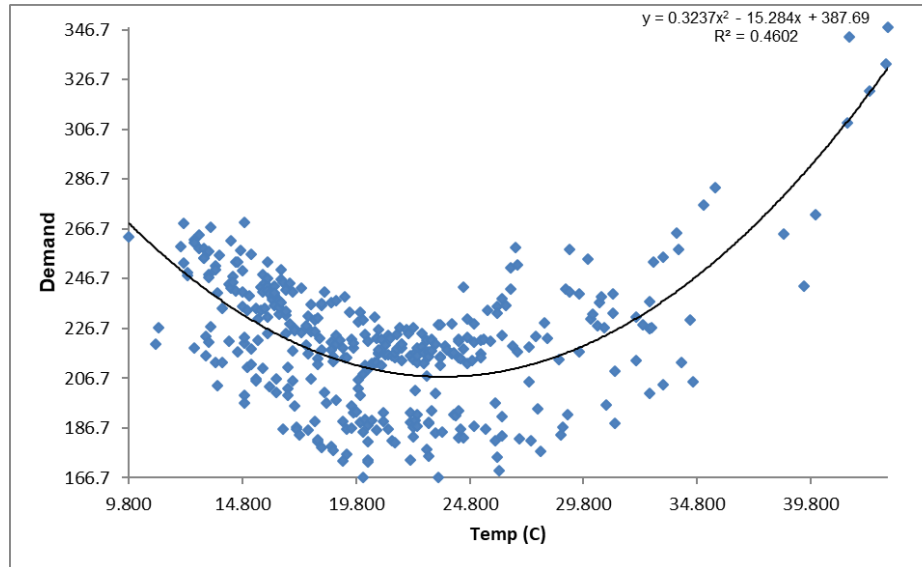
# Example 3: Daily Electricity Demand with Temperature and Work Day Predictors – Run Charts



SigmaXL > Time Series Forecasting > Run Chart

Example 3: Daily Electricity Demand with Predictors – ElecDaily.xlsx  
Victoria, Australia, 2014.

# Example 3: Daily Electricity Demand with Temperature and Work Day Predictors – Scatterplot and Box Plot



SigmaXL > Graphical Tools > Scatterplots

SigmaXL > Graphical Tools > Boxplots

Example 3: Daily Electricity Demand with Predictors – ElecDaily.xlsx  
Victoria, Australia, 2014.

# Example 3: Daily Electricity Demand with Temperature and Work Day Predictors – ARIMA Forecast with Predictors

ARIMA with Predictors Forecast

Numeric Time Series Data (Y) >>

Demand

OK >>

Optional Time Axis Labels >>

Date

Cancel

Optional Continuous Pred. (X) >>

Temp (C)  
TempSq

Help

Optional Categorical Pred. (X) >>

WorkDay

<< Remove

No. of Forecast Periods 24

Prediction Interval 95.0 %

☐ Specify Model Periods

Start Model at Period 1

☒ Withhold Periods 0

☐ End Model at Period

Model Options

☒ Seasonal Frequency

☐ Specify 12

☒ Select 7 - Daily

☐ Automatically Detect

☒ Display ACF/PACF/LB Plots

☒ Display Residual Plots

☐ Box-Cox Transformation

☒ Rounded Lambda

☐ Optimal Lambda

☐ Lambda & Threshold (Shift)

ARIMA Model Options

☒ Automatic Model Selection

☐ Specify Model

OK >>

Cancel

Help

☒ Stepwise Procedure

☐ Extended Model Search. Time limit 300 seconds.

Model Selection Criterion

☒ AICc - Akaike information criterion with small sample size correction

☐ AIC - Akaike information criterion

☐ BIC - Bayesian information criterion

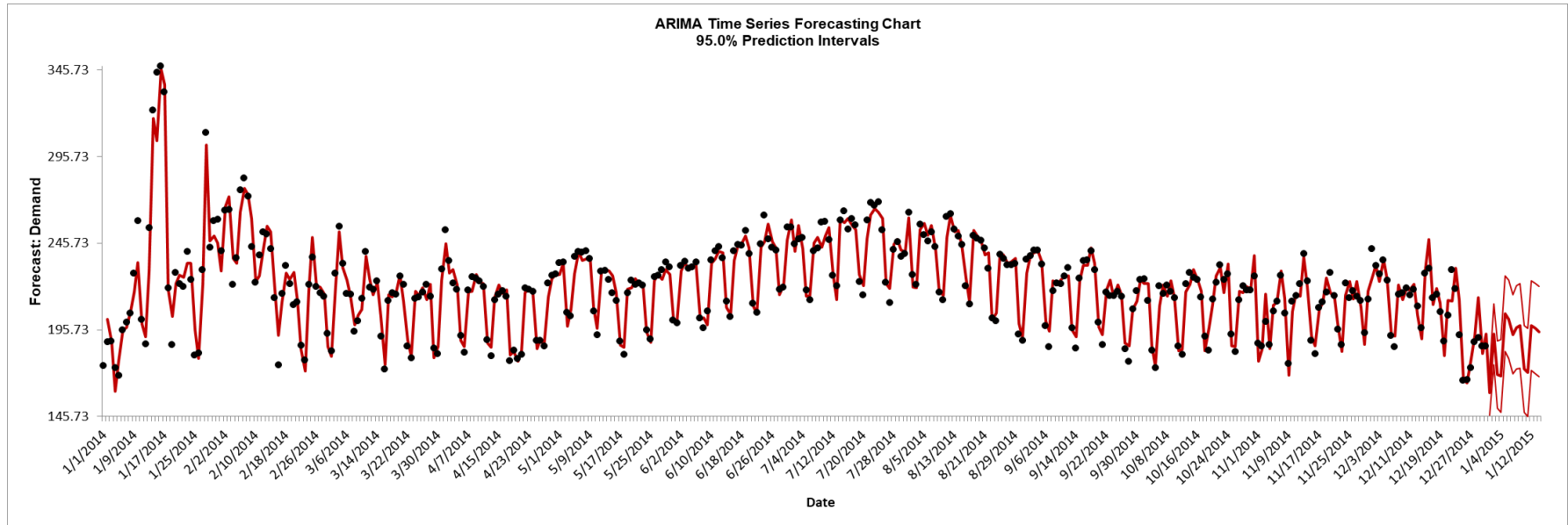
☐ Specify Nonseasonal Differencing (d) 0

☐ Specify Seasonal Differencing (D) 0

SigmaXL > Time Series Forecasting > ARIMA Forecast > Forecast with Predictors

Example 3: Daily Electricity Demand with Predictors – ElecDaily.xlsx,  
Sheet “Forecast 2 Weeks”.

# Example 3: Daily Electricity Demand with Temperature and Work Day Predictors – ARIMA Forecast with Predictors

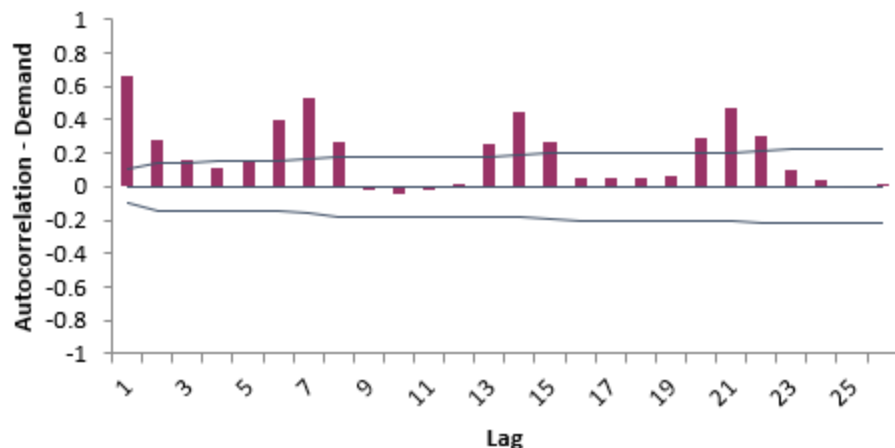


ARIMA Model Summary	
AR Order (p)	2
I Order (d)	1
MA Order (q)	2
SAR Order (P)	2
SI Order (D)	0
SMA Order (Q)	0
Seasonal Frequency	7
Include Constant	0
No. of Predictors	3
Model Selection Criterion	AICc
Box-Cox Transformation	N/A
Lambda	
Threshold	

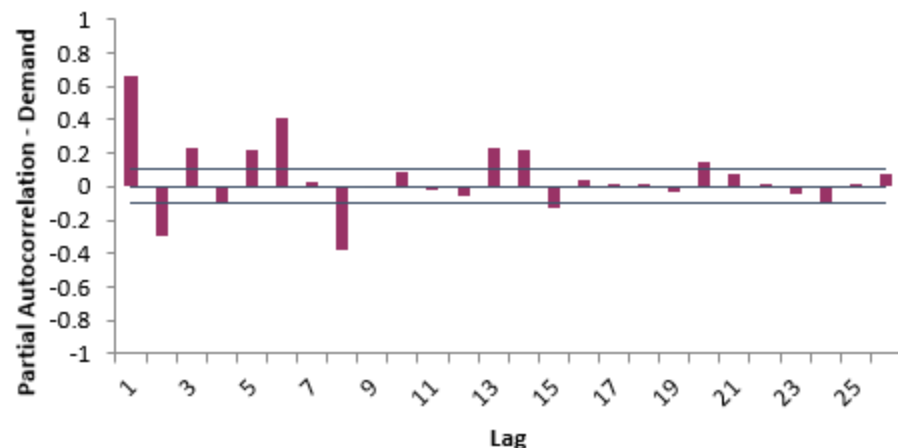
Parameter Estimates				
Term	Coefficient	SE Coefficient	T	P
AR_1	-0.063223451	0.075658448	0.83564	0.4039
AR_2	0.673128346	0.067270503	10.0063	0.0000
MA_1	0.022660844	0.043288704	0.52348	0.6010
MA_2	0.929862871	0.039474102	23.5563	0.0000
SAR_1	0.200902989	0.053912363	3.72647	0.0002
SAR_2	0.402632085	0.05676416	7.09307	0.0000
Temp (C)	-7.501559029	0.446098708	16.8159	0.0000
TempSq	0.17890261	0.008530253	20.9727	0.0000
WorkDay_1	30.56943168	1.295720007	23.5926	0.0000

# Example 3: Daily Electricity Demand with Temperature and Work Day Predictors – ARIMA Forecast with Predictors

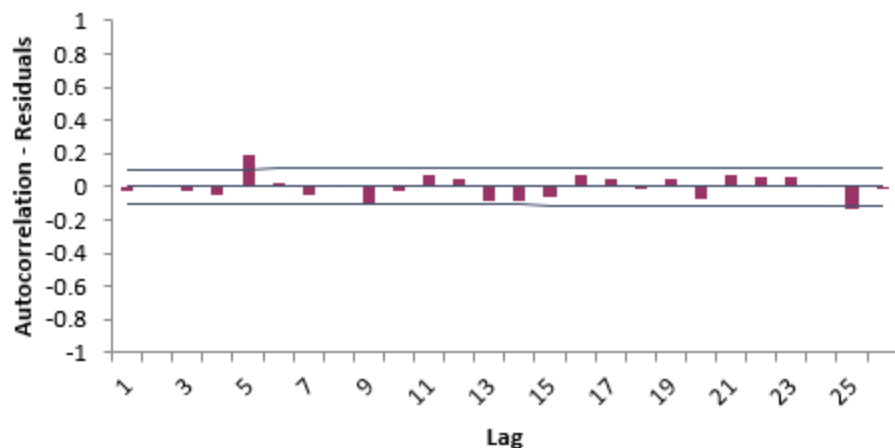
Autocorrelation Function (ACF) Plot  
Significance Limit Alpha = 0.05



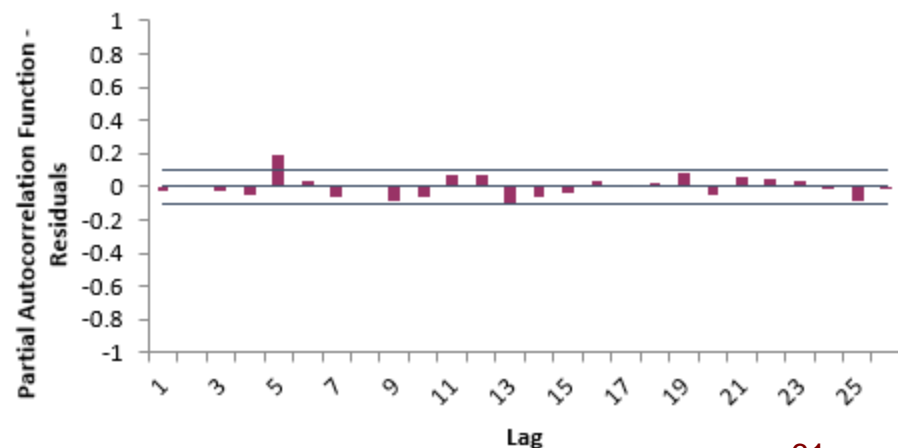
Partial Autocorrelation Function (PACF) Plot  
Significance Limit Alpha = 0.05



Autocorrelation Function (ACF) Plot - Residuals  
Significance Limit Alpha = 0.05



Partial Autocorrelation Function (PACF) Plot - Residuals  
Significance Limit Alpha = 0.05



# Example 3: Daily Electricity Demand with Temperature and Work Day Predictors – ARIMA Control Chart with Predictors

ARIMA with Predictors Control Chart

**Numeric Time Series Data (Y) >>** Demand **OK >>**  
**Optional Time Axis Labels >>** Date **Cancel**  
**Optional Continuous Pred. (X) >>** Temp (C)  
TempSq  
**Optional Categorical Pred. (X) >>** WorkDay  
**<< Remove**

☐ **Specify Model Periods**  
Start Model/Control Limit Calculations at Period 1  
☒ **Withhold Periods** 0  
End Model/Control Limit Calculations at Period  
☐ **Limit Calculations at Period**

**Model Options**  
☒ **Seasonal Frequency**  
☐ **Specify** 12  
☒ **Select** 7 - Daily  
☐ **Automatically Detect**

☐ **Display ACF/PACF/LB Plots**  
☐ **Display Residual Plots**  
☐ **Box-Cox Transformation**  
☒ **Rounded Lambda**  
☐ **Optimal Lambda**  
☐ **Lambda & Threshold (Shift)**

ARIMA Model Options

☒ **Automatic Model Selection** **OK >>**  
☐ **Specify Model** **Cancel**  
**Help**

☒ **Stepwise Procedure**  
☐ **Extended Model Search. Time limit** 300 **seconds.**

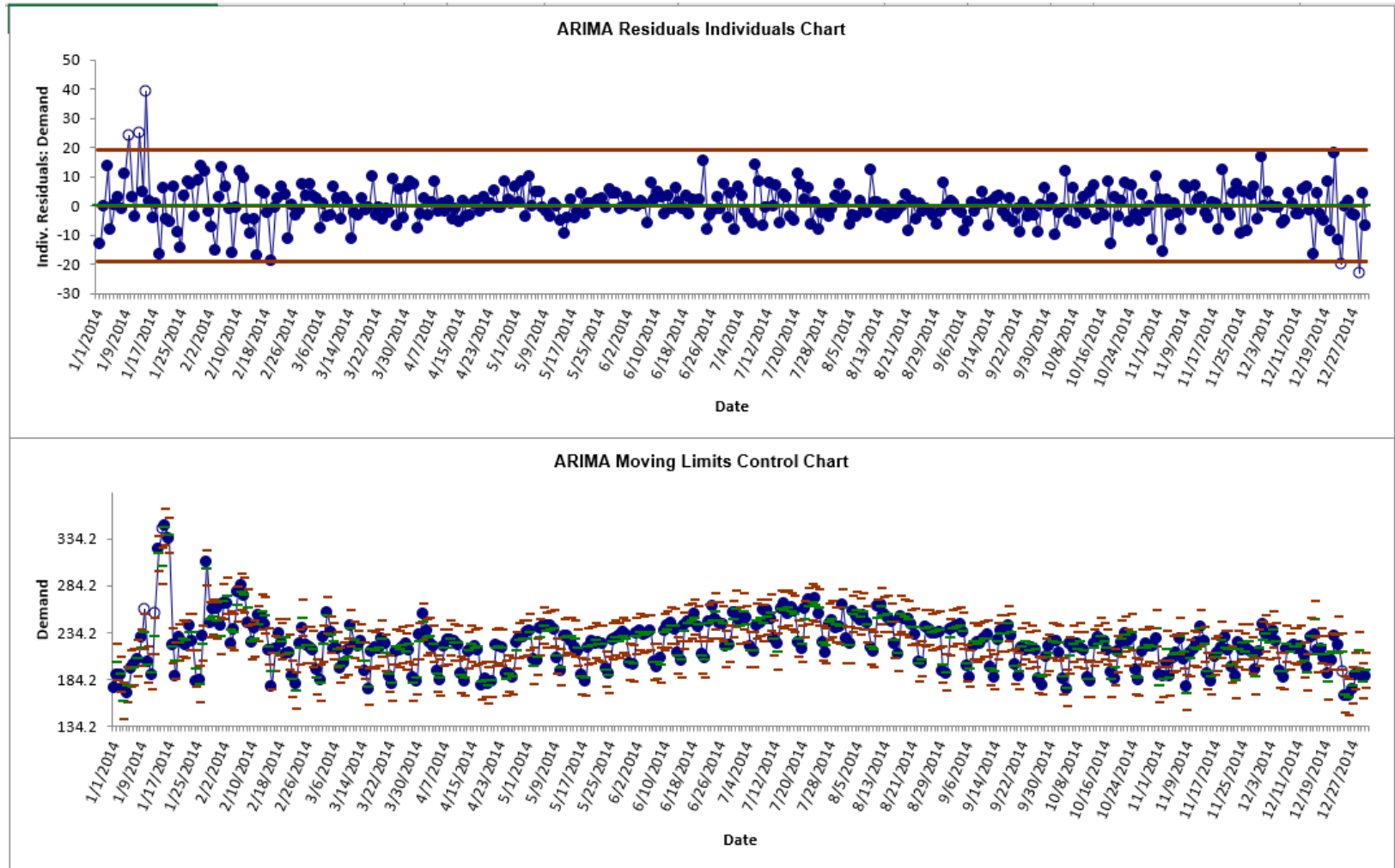
**Model Selection Criterion**  
☒ **AICc - Akaike information criterion with small sample size correction**  
☐ **AIC - Akaike information criterion**  
☐ **BIC - Bayesian information criterion**

☐ **Specify Nonseasonal Differencing (d)** 0  
☐ **Specify Seasonal Differencing (D)** 0

SigmaXL > Time Series Forecasting > ARIMA Control Chart > Control Chart with Predictors

Example 3: Daily Electricity Demand with Predictors – ElecDaily.xlsx, Sheet 1.

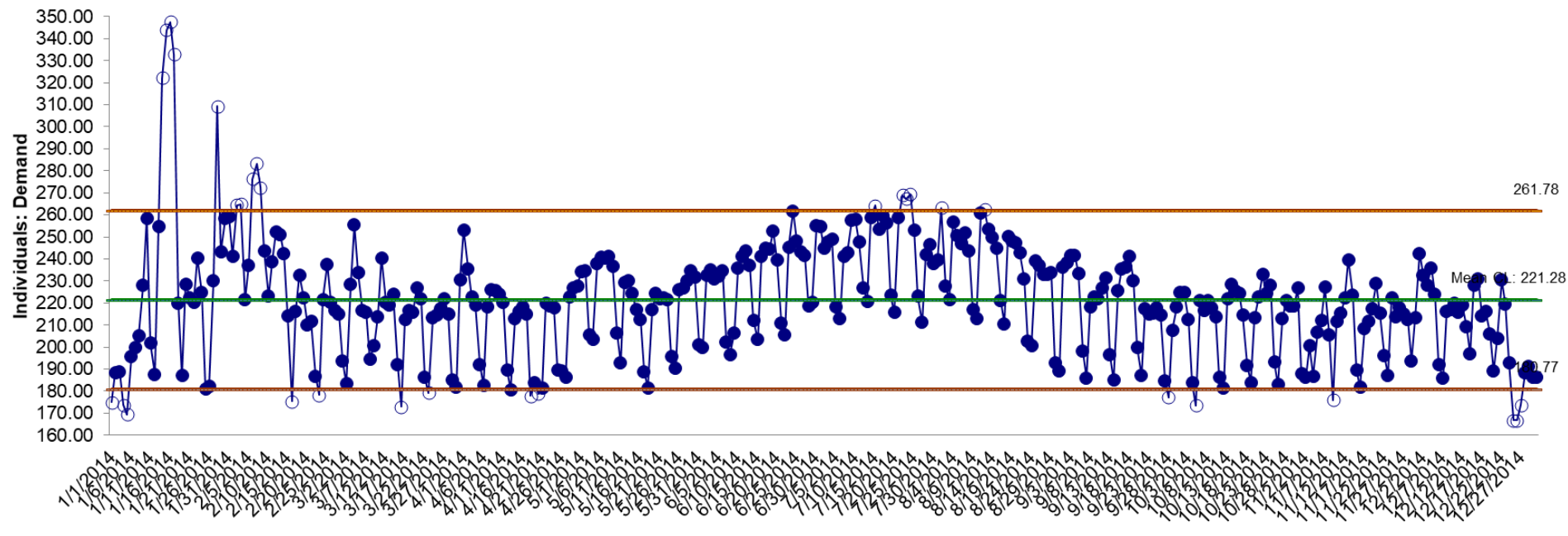
# Example 3: Daily Electricity Demand with Temperature and Work Day Predictors – ARIMA Control Chart with Predictors



ARIMA Model: Demand - Model Automatically Selected

Model Periods: All observations are used in the ARIMA model estimation. No withhold periods available for out-of-sample forecast accuracy evaluation.

# Example 3: Daily Electricity Demand with Temperature and Work Day Predictors - Individuals Control Chart for Raw Data



Note the additional out-of-control signals.

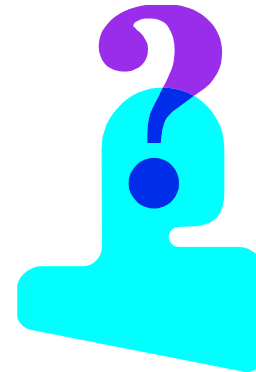
SigmaXL > Control Charts > Individuals



# What's New in SigmaXL® Version 9

## Part 3 of 3: Control Charts for Autocorrelated Data

Questions?



# References

1. Alwan, L.C., and Roberts, H.V. (1988), “Time Series Modeling for Statistical Process Control,” **Journal of Business and Economic Statistics**, 6, 87-95.
2. Box, G. E. P. , Jenkins, G. M. , Reinsel, G. C. and Ljung, G.M. (2016). *Time Series Analysis, Forecasting and Control*, 5<sup>th</sup> edition, Wiley.
3. Hunter, J.S. (1986), “The Exponentially Weighted Moving Average,” **Journal of Quality Technology**, 18, 203-210.
4. Hyndman, R.J., & Athanasopoulos,G. (2018). *Forecasting: principles and practice*, 2<sup>nd</sup> edition, OTexts: Melbourne, Australia. OTexts.com/fpp2.
5. Hyndman, R. J. and Y. Khandakar, (2008). “Automatic Time Series Forecasting: The forecast Package for R. ” **Journal of Statistical Software**, 27(3), 1-22.

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6. Montgomery, D. C., and Mastrengelo, C.M. (1991), “Some Statistical Process Control Methods for Autocorrelated Data,” **Journal of Quality Technology**, 23, 179-204.
7. Montgomery, D. C. (2013). *Introduction to Statistical Quality Control*, 7<sup>th</sup> edition, Wiley.
8. Montgomery, D. C., Jennings, C.L., and Kulahci, M. (2015). *Introduction to Time Series Analysis and Forecasting*, 2<sup>nd</sup> edition, Wiley.
9. NIST/SEMATECH e-Handbook of Statistical Methods, <https://www.itl.nist.gov/div898/handbook>.
10. Woodall, W.H. and Faltin, F.W. “Autocorrelated Data and SPC” **ASQC Statistics Division Newsletter**, 13(4).